The Mathematics Teacher

DECEMBER 1958

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A. E. MEDER, JR.

The Seventh Mathematical Olympiad for secondary school students in Poland
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GLEN D. VANNATTA

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What is wrong with Euclid?

A. E. MEDER, JR., *Rutgers University, New Brunswick, New Jersey.*

Euclid's achievement will always be remembered, but should our high school course in geometry be based on Euclid's postulates? Isn't it possible that with today's knowledge of mathematics a better set of postulates could be found?

ABOUT FIFTY YEARS AGO the great geometer, Felix Klein, devoted a sizable section of his famous course of lectures, *Elementary Mathematics from an Advanced Standpoint,* to a critical discussion of Euclid's *Elements.*[1] Now that groups like the Commission on Mathematics and others interested in revising the secondary school mathematics curriculum are urging that schools depart from the traditional Euclidean organization of geometry, there would seem to be some value in pointing out the way in which these present-day suggestions were foreshadowed by an advanced thinker a half-century ago.

Although Euclid's development of geometry dominated instruction in this subject for nearly two millenia, proposals are being made not only in this country but in Europe as well that at long last this treatment of geometry be abandoned, or at least greatly modified. What is wrong with Euclid, anyhow?

Euclid must be judged on the basis of his own objectives. Did he intend to write a text for school boys and girls? Did he intend to present a full treatment of geometry as known to the Greeks? Did he intend to write a scientific treatise?

EUCLID'S PURPOSE

Euclid intended, said Klein, to write an introduction to the study of geometry, and therefore, a fortiori, an introduction to the study of mathematics itself, from a particular point of view. This was to provide an introduction to mathematics in a manner that would serve acceptably as a preparation for general philosophical studies "in the sense of Plato."

To accomplish this purpose, practical applications of geometry were pushed aside and emphasis was placed on deduction and logic, not on geometric thinking. The subject matter treated was strictly limited. Conic sections, concerning which a great deal was known by Greek mathematicians of Euclid's day, were not even mentioned by Euclid.

EUCLID AND ARCHIMEDES

To show that Euclid did not truly reflect in full measure the high accomplishments of Greek mathematics, but rather had the philosophic orientation suggested above, Klein compared his work with that of Archimedes, by common consent the greatest mathematician of antiquity.

Archimedes had at least four characteristics not revealed in Euclid's work. First, he had a taste for calculation: he gave an excellent approximation to the value of π. Second, he showed an extensive interest in the applications of mathematics. Third, he was a creator and pioneer. Fourth, he proceeded genetically, or, as we would now be apt to say, heuristically, emphasizing the chain of thought by which a mathematical result is discovered,

[1] Felix Klein, *Elementary Mathematics from an Advanced Standpoint,* II (New York: The Macmillan Co., 1939), 188–208.

rather than a formal proof of the result.

Euclid, on the other hand, had no interest in calculation: he contented himself with showing that the area of a circle is proportional to the square on its radius, but did not compute the factor of proportionality, that is, π. Similarly, he was content to postulate abstractly that a line may be drawn through two points, or a circle about a point, but had nothing to say about how these postulated constructions were to be performed: he did not even mention the instruments of classical construction, ruler and compass. Moreover, Euclid was interested in the collection and systematization of results, not in their discovery, and his treatment gave no clue to the underlying mathematical thought-patterns involved. It emphasized instead only a kind of rigid separation of the presentation of results into a formal classification: Proposition, Hypothesis, Proof, Conclusion.

Thus far our criticism of Euclid has been based on a comparison of his procedures with those of another mathematician of antiquity. Let us now look at his work from the viewpoint of the present day. In doing this, we would expect to find a number of respects in which modern mathematics has surpassed the achievements of the Greeks; this is indeed the case. It is not to the discredit of Euclid, then, that we must indict him, ex post facto, to be sure, on the following counts.

"Ersatz" calculations

First, since the Greeks had no adequate arithmetic or algebra, and in particular neither decimal fractions nor algebraic symbolism, it was necessary for Euclid to use a substitute—calculation in geometric form. It is interesting to note that in the original German of his lectures fifty years ago, Klein[2] described Euclid's method of calculation as "Ersatz," the same word that became familiar to us all during World War II as describing an inferior wartime substitute for an unobtainable commodity. This "Ersatz" method of geometric calculation in lieu of an adequate algebra meant, of course, that neither negative nor imaginary numbers could be utilized, and this in turn led to the necessity for a long enumeration of special cases in lieu of a general method. Through no fault of his own, but because of the inadequacy of Greek algebra, Euclid stands indicted, from the viewpoint of the present day, of using a clumsy geometric method of calculation.

Separation of plane and solid geometry

Second—and this is his fault—we note that Euclid's *Elements* embodies a rigidly separate treatment of plane and solid geometry. This Felix Klein felt, a half-century ago, to be completely out of the spirit of modern mathematics.

Logical defects

Third, and this is the most serious charge of all, Euclid failed seriously in his admittedly notable ideal of seeking to present a logically unimpeachable treatment of geometry.

His first failure is in respect to his definitions. He completely failed to realize that it is impossible, without circularity, to define every term. He did in fact attempt, unsuccessfully of course, to define everything. "A point is that which has no parts." "A line is length without breadth." These so-called definitions do not in fact set forth properties that distinguish the entities being defined from all others.

He failed also in respect to his postulates. He formally postulated some things, but omitted any mention of others that are equally necessary. For example, there were postulated the possibility of drawing a line through two points, of prolonging a line segment, and of describing a circle with a given center passing through a given point. But it was not postulated that two circles intersect if each passes through

[2] *Ibid.*, 3d German edition (Berlin: Julius Springer, 1925), p. 209.

the center of the other, although this fact is used, and cannot be established on the basis of the postulates laid down by Euclid.

Difficulties with superposition

Again, the treatment of the congruence proofs (for triangles) was defective. Euclid went to a great deal of trouble to show that a line segment AB can be laid off on another line CD with a specific point C as initial point of the segment, presumably in order that the superposition of $A'B'$ on AB (in the case of two triangles, ABC and $A'B'C'$, with $AB=A'B'$, $BC=B'C'$, and angle $ABC=$ angle $A'B'C'$) will be meaningful. Yet he failed utterly to give any similar treatment of what is to be understood when one angle is superposed on another.

It is important to pause here to look into the matter of superposition more carefully. Why should Euclid be condemned for his failure to explain the superposition of angles? Should we not rather inquire why he should have gone to all that trouble to explain what was meant by superposition of segments? After all, isn't the notion of picking up a triangle ABC and putting it down on a triangle $A'B'C'$ perfectly clear intuitively?

To this we must answer that of course if we are dealing with physical triangles made of wire or wood or even of chalk lines, this notion is clear. But if we are dealing with abstract conceptual entities whose properties exist only because they have been postulated or logically derived from postulated properties, we most certainly do not have an intuitive notion of superposition. To suppose that we do is to confuse the models or figures we use to help our intuition with the logical constructs about which we are proving our theorems.

The abstract conceptions of geometry have no physical existence; they exist only in our minds. Can anyone presume to say that he has an intuitive notion of how to superpose one idea upon another? Or if it be granted that he has, can there be any presumption that his notion is necessarily the same as anyone else's notion?

No, Euclid was absolutely right in explaining in detail with the utmost care how to superpose two segments, and what precisely is meant by this language. His failure to do the same for angles clearly vitiates the whole method of proof by superposition. As far as a proof is concerned, we cannot introduce intuition, or physical properties of a model or figure, or anything else that is not derived logically from our underlying axioms. This is what superposition proofs in Euclid inevitably do.

Bertrand Russell has pointed out[3] that the matter is even worse than this, that in fact there is an inherent contradiction in the process of defining equality or congruence by superposition. For superposition presupposes that what is superposed is rigid, and does not change its size or shape in the process of being superposed; but this is precisely what we are trying to prove by means of superposition, namely, that two figures have the same size and shape. We therefore have an inherent vicious circle when we try to establish congruence by means of superposition unless we have established unimpeachable logical meaning derived from the axioms for this procedure.

It was undoubtedly for these reasons that Felix Klein observed that the difficulties with the superposition proofs are so real and so significant that after encountering them one could no longer speak of even the possibility that Euclid might have attained his ideal of an unimpeachable logical treatment of geometry.

Assumptions of order

Finally, we must call attention to perhaps the most serious omission of all. This is that Euclid made no mention of the order of points on a line, or of the concept of "betweenness." That is to say, we are

[3] Bertrand Russell, *Principles of Mathematics* (2d ed.; New York: W. W. Norton & Co., Inc., 1938), p. 161.

Figure 1

utterly unable, on the basis of Euclid's formulation of geometry, to speak of point B as between A and C in the accompanying figure. (See Fig. 1.)

It is this omission that leads to the familiar "fallacy" that every triangle is isosceles. It will be recalled that this particular "proof" is obtained by drawing a figure in which the perpendicular bisector of the base of a triangle and the bisector of the opposite angle seem to meet inside the triangle. (See Fig. 2.)

Draw $OF \perp AC$ and $OE \perp BC$.

Since $\angle OCF = \angle OCE$, we get $\triangle OCF \cong \triangle OCE$, $CF = CE$, and $OF = OE$.

Also, $AO = BO$ and we get $\triangle AOF \cong \triangle BOE$, and $FA = EB$.

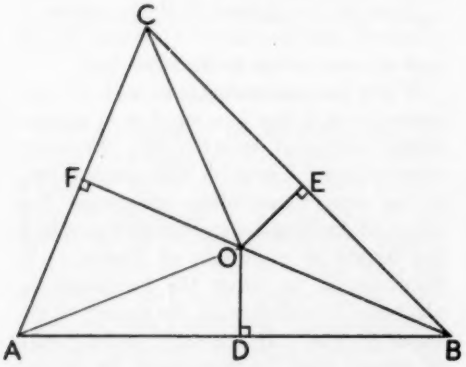
Therefore

$$CA = CF + FA = CE + EB = CB.$$

Actually, point O is always outside the triangle, although on the basis of the Euclidean formulation of geometry this fact cannot be proved. If the figure is correctly drawn (Fig. 3), $CF = CE$, $FA = EB$. But it does not follow that $AC = BC$.

It is perhaps worth pointing out what the connection is between the concept of "betweenness" and the concepts of "inside" and "outside" as applied to the triangle. Let K be the point in which the bisector of angle C meets AB. Then to say that O is outside the triangle is the same as saying that K is between A and D, while to say that O is inside the triangle is the same as saying that D is between A and K or, in other words, that K is not between A and D.

Figure 2



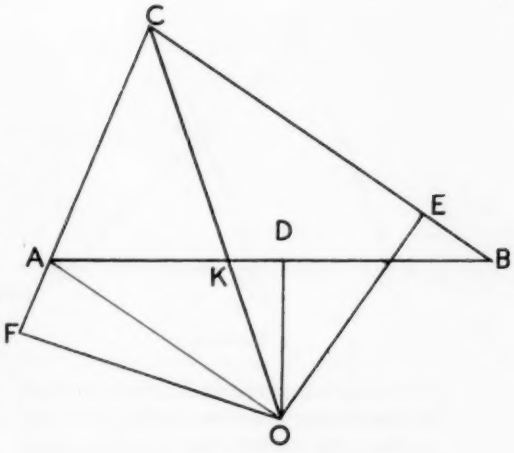


Figure 3

It is apparent from our discussion that the omission of assumptions concerning "betweenness" or order of points in a line is closely connected with the inadequate algebra from which Greek mathematics suffered. If line segments had had signs or directions in Greek geometry, it is unlikely that so serious a defect in the logical foundations of the Euclidean treatment of geometry would have passed unnoticed.

Incidentally, this discussion indicates also why and how we get along without "order" or "betweenness" concepts. We introduce them unnoticed, as a matter of fact, when we bring in the number line in algebra. The order of points is thus founded on the order of numbers; to us, having an adequate algebra, the latter is so familiar that the former is also accepted without question.

It is, of course, possible to correct the defect by explicitly adding additional

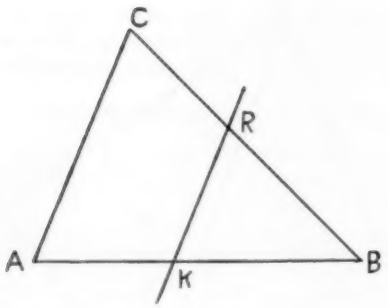


Figure 4

assumptions to those of Euclid. The first mathematician to do this was Moritz Pasch[4] in 1882, who added what is often called Pasch's Axiom: "If a line intersects one side of a triangle, it intersects one of the other two sides also." (See Fig. 4.)

Again, the idea of betweenness is implicitly involved here. For if a line L intersects one side of a triangle, say AB, in a point K, this is to say that K is between A and B. Then the axiom of Pasch says that there is a point R on line L such that R is between B and C or between A and C.

To show more fully the connection between "inside" and "outside," "betweenness" and the axiom of Pasch would carry us too far afield. It should be clear, however, that the notion of order of points on a line is involved in all of these concepts.

Summary

Let us then sum up. What is wrong with Euclid? In one sense, only that he was a man of his times. Modern mathematics has surpassed Greek mathematics; Euclid's treatment of geometry is therefore inadequate for the present day.

But it is possible to be more specific. There are at least three specific things wrong with Euclid:

1. His development of geometry contains many serious omissions. Most significant is the failure to introduce any concept of order among points on a line. Others are the failure to postulate or prove certain constructions actually used and the confused treatment of congruence of triangles.

2. The development in general is unnecessarily cumbersome, because Euclid had only an imperfect algebra available to him.

3. The overemphasis on logic interferes with the development and understanding of geometric relationships.

Euclid's remarkable insights

In conclusion, having emphasized the shortcomings of Euclid, it is only fair to call attention to three outstanding, even remarkable, insights; to three things that are emphatically right with Euclid:

First, his tremendous ideal of establishing geometry on an unimpeachable logical foundation. The great historical significance of Euclid's *Elements* that will forever remain is that through this book that ideal was transmitted to future generations of mathematicians. Though Euclid failed to attain the ideal, he conceived it. Future mathematicians, notably David Hilbert, completed the task Euclid had set.

Second, his recognition of the importance of the parallel postulate and the necessity of assuming it. Many later mathematicians attempted unsuccessfully to prove it, and not until the nineteenth century was Euclid's judgment on this point finally vindicated.

Third, his recognition of the importance—indeed, the necessity—of laying down such an assumption as the following:

If any line segment AB be laid off successively on a line l, a number m always exists such that $m \cdot AB > AX$, whatever point on line l X may be. This assumption, or an equivalent, often misnamed the axiom of Archimedes and sometimes called the axiom or postulate of Eudoxus, is fundamental to either the geometric or algebraic formulation of the theory of rational numbers. It is greatly to the credit of Euclid that he recognized its funda-

[4] Moritz Pasch, *Vorlesungen über neuere Geometrie* (Leipzig: B. G. Turner, 1882) (reprinted Ann Arbor, Michigan: J. W. Edwards, Publisher, Inc., 1945), p. 20.

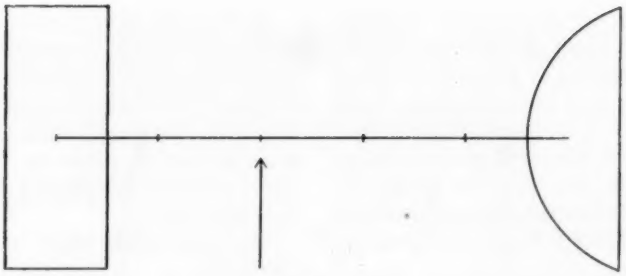


Figure 5

mental importance. It is by virtue of this assumption that "absolute infinitesimals" are excluded from mathematics.

Classroom implications

Finally, what are the implications for the classroom of this analysis of the shortcomings of Euclid? In a word, simply that we must no longer be dominated by a treatment of geometry that never was intended to produce mathematicians, but philosophers; that overemphasizes logical formalism rather than stressing mathematical understanding; that is marred by serious omissions; that is unnecessarily cumbersome because of difficulties that we have long known how to avoid or resolve.

We can also note that these aims of Euclid indicate that he was writing for mature scholars, not schoolboys. It is unlikely that a discussion with such objectives can be made suitable for school use by mutilating it, or watering it down. Yet this has too often been attempted. Both Euclid and the schoolboys deserve better treatment.

Discovery and power

Our aim should be to encourage our students to approach the study of geometry in a spirit of discovery and adventure, to lead them to understand to the fullest extent consistent with their ability and maturity how mathematical facts are discovered and how mathematical concepts are formulated and may be generalized and extended. In short, we should try to lead our students to develop mathematical understanding and power.

In one of the extant manuscripts of Archimedes, for example, are some notable examples of how this may be done. We cite one briefly, merely to clarify our meaning. In attempting to find the formula for the area of a segment of a parabola, Archimedes imagines this area placed at one end of a lever arm and at the other end the rectangle circumscribing the parabola. (See Fig. 5.) He then undertakes to determine where the fulcrum must be placed so that the figure may be in equilibrium. From these considerations he deduces that the area of the parabolic segment is two-thirds that of the rectangle.

How heuristic this is, but how foreign to the spirit of Euclid!

Deductive reasoning

Of course, we will also make every effort to see to it that our pupils understand the nature of deductive reasoning, and can put together an acceptable mathematical proof. Geometry has much to contribute to the development of this understanding. Even Euclid, with all his shortcomings, has something worthwhile to contribute, as was pointed out above. But so also have algebra, arithmetic, trigonometry, and analytic geometry. No longer does it seem reasonable to devote as much time and attention as heretofore to a development on which we can so markedly improve.

We must also be sure that, as we seek to develop an understanding of deductive

reasoning, we do not permit excessive formalism to crowd out the joy of personal discovery. For it is one of the great values of geometry that this subject is at once so simple that practically any student can have the thrill of discovering some geometric fact, however trivial, for himself, and so complex that problems can be set that will tax the powers of the most able.

In short, in our instruction we should remember that the characteristics of the mathematician are Imagination, Conception, and Generalization, and that Reasoning, in the words of a philosopher of a bygone generation, is but the smooth pavement on which the chariot of the mathematician rolls.

Let Archimedes rather than Euclid be our guide.

What's new?

BOOKS

Secondary

The High School in a New Era, edited by Francis S. Chase, Harold A. Anderson. Chicago: University of Chicago Press, 1958. Cloth, xiv+465 pp. $5.75.

Plane Geometry (rev. ed.), Royal A. Avery, William C. Stone. Boston: Allyn and Bacon, Inc., 1958. Cloth, ix+498 pp., $3.96.

Teachers' Manual for Plane Geometry, A. M. Welchons, W. R. Krickenberger, Helen R. Pearson. Boston: Ginn and Company, 1958. Paper, iv+149 pp., $1.28.

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College

Elementary Number Theory, Edmund Landau (translated by Jacob E. Goodman). New York: Chelsea Publishing Company, 1958. Cloth, 256 pp., $4.95.

Engineering Mathematics, Robert E. Gaskell. New York: Henry Holt and Company (Dryden Press), 1958. Cloth, xvi+462 pp., $7.25.

Functions of Real Variables, William Fogg Osgood. New York: Chelsea Publishing Company, 1958. Cloth, xii+262 pp., $4.95.

Miscellaneous

Computability and Unsolvability, Martin Davis. New York: McGraw-Hill Book Company, Inc., 1958. Cloth, xxv+210 pp., $7.50.

The Exploration of Time, R. N. C. Bowen. New York: The Philosophical Library, 1958. Cloth, vii+144 pp., $6.00.

History of Mathematics, Volume I (Dover republication), David Eugene Smith. New York: Dover Publications, Inc., 1958. Paper, xxii+596 pp., $2.75 ($5.00, set).

History of Mathematics, Volume II (Dover republication), David Eugene Smith. New York: Dover Publications, Inc., 1958. Paper, xii+725 pp., $2.75 ($5.00, set).

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101 Puzzles in Thought and Logic, C. R. Wylie, Jr. New York: Dover Publications, Inc., 1958. Paper, 115 pp., $1.00.

Operational Mathematics (2d edition), Ruel V. Churchill. New York: McGraw-Hill Book Company, Inc., 1958. Cloth, ix+337 pp., $7.00.

The Philosophy of Humanism (4th ed.), Corliss Lamont. New York: Philosophical Library, 1949. Cloth, xi+243 pp., $2.75.

The Principles of Science—A Treatise on Logic and Scientific Method (Dover Republication), W. Stanley Jevons. New York: Dover Publications, Inc., 1958. Paper, liii+786 pp., $2.98.

The Seventh Mathematical Olympiad for secondary school students in Poland[1]

IZAAK WIRSZUP, *University of Chicago, Chicago, Illinois.*

It is important for Americans to know what is expected of the Soviet Union's gifted mathematics students. Teachers will find challenging problems in this summary of the Polish Olympiad (1955–1956).

THE SOVIET MODEL

IN ADDITION TO THE COMPREHENSIVE and compulsory in-school training, there has been an abundant amount of thought and planning by Soviet educators to produce a supplementary program of extracurricular mathematical activities designed to encourage and aid the gifted student to reach still higher levels of mathematical skill and understanding during his school years.

One of the means for stimulating student interest and developing student ability is the publication of enrichment literature.[2] Prominent Soviet mathematicians and teachers have produced a vast array of books and articles to supplement the school programs. The more popular and outstanding of these books are published in editions of hundreds of thousands and are read by teachers and students alike.

The major vehicle for utilizing this enrichment literature is the class mathematics club. From the fifth grade[3] on, the more successful and talented students are encouraged to join their class mathematics club. These clubs normally meet one evening every two weeks and consist of approximately fifteen members each.

An even higher level of extracurricular activities is available to secondary school students in the cities which have universities or polytechnic institutes. Once a week throughout the academic year[4] these institutions of higher learning hold their "School mathematics circle." Large audiences of young pupils hear research mathematicians give lectures ranging from elementary mathematics from a higher standpoint to complex topics of modern mathematics. This is in addition to their regular class mathematics club.

But the high points of extracurricular activity for secondary school students are the *mathematical olympiads.* Mass problem-solving contests are held yearly in almost every city of the Soviet Union. There is wide public interest in these events, and successful students gain considerable prestige. The first such olympiad was organized by the University of Leningrad in 1934.

[1] This paper is part of a Survey of Recent East European Literature in Intermediate Mathematics. The Survey is being conducted by the College Mathematics Staff of the University of Chicago and is sponsored by the National Science Foundation.

[2] See Izaak Wirszup, "Some Remarks on Enrichment," THE MATHEMATICS TEACHER, XLIX (November 1956), 519–527.

[3] The fifth-grade club (age 11–12) is usually called "The club of the gay mathematicians."

[4] At the University of Kiev, for instance, the secondary school circle meets every Sunday from 10:00 to 12:00 A.M.

Today thousands of students enter these events every year.

The primary educational function of these contests is to stir up scientific enthusiasm and mathematical curiosity among youngsters. Moreover, they force them to think rigorously and to express themselves clearly and simply. As one Soviet educator stated, the problems are constructed "to make the student think much and write little." Furthermore, the olympiads fortify the confidence of the bright students in their own abilities while simultaneously emphasizing the fact that stubborn work is the only sure way to success in mathematics.

In addition to these indisputable benefits to the students, the events have great practical value to the higher institutions. The olympiads serve to discover the gifted students who merit increased attention and support by the universities. The slogan of the educational authorities is, "Not a single mathematical talent must be wasted nor left unnoticed." Needless to say, many former winners are now established scholars.[5]

The Polish Mathematical Olympiad

Poland, with its distinguished school of mathematical research and strong tradition of mathematical education, has not neglected to apply the ideas and successful experiments of the Soviets to its own students. Since 1950 it has held annual *national* olympiads in mathematics, supervised by the national Ministry of Education and outstanding Polish mathematicians.

The Seventh Polish Mathematical Olympiad for secondary school students was held in 1955–1956.[6] The Executive Committee of the Olympiad was appointed by the Ministry of Education and was chaired by Professor Stefan Straszewicz;[7] the Director of the Olympiad was Professor K. Zarankiewicz. The membership included, among others, the Director of the Mathematics Section in the Central Office of Improvement of Educational Cadres, a representative of the Ministry of Education, a representative of the Central Office of Professional Training, and Professor E. Otto. This committee held 12 meetings.

The Polish Mathematical Society, working in conjunction with the Executive Committee of the Olympiad, organized seven regional committees from the districts of Warsaw, Kraków, Wrocław, Poznań, Toruń, Lublin, and Łódź. The chairman of the Warsaw Regional Committee was Professor W. Sierpiński, dean of Polish mathematicians and distinguished scholar. The members of the committee included Professors M. Czyżykowski, S. Kulczycki, A. Białaś, and others. Other regional committees had, as members, Professors J. Jodłowski and J. Szarski (Kraków), B. Iwaszkiewicz (Wrocław), W. Jankowski, J. Albrycht, A. Alexiewicz, and Z. Butlewski (Poznań). These committees met 54 times in conducting the Olympiad.

The Olympiad consisted of three rounds. The *first (preparatory) round* was held in the local schools during the period from October 1, 1955 to January 10, 1956.

Below is a distribution of the total number of the 1642 contestants of the first round:

General-education schools

Boys	969
Girls	410

1379 students representing 324 schools

Semiprofessional schools

Boys	234
Girls	29

263 students representing 70 schools

[5] For more information on the Soviet mathematical olympiads, see B. V. Gnedenko, "Mathematical Education in the U.S.S.R.," *The American Mathematical Monthly*, LXIV (June–July 1957), 389–408.

[6] The account presented below is derived from the *Report of the Executive Committee of the Seventh Mathematical Olympiad*, published by the Ministry of Education, Warsaw, 1957.

[7] Professor Stefan Straszewicz of Warsaw University and of the Warsaw Polytechnic Institute is one of the leading lights of the Polish olympiads. His books and papers have powerfully influenced the development of secondary mathematical education in Poland since 1920.

The following 12 problems were given in the first round.

PROBLEMS OF THE FIRST (PREPARATORY) ROUND[8]

1. Solve the following system of equations:

$$(x^3+y^3)(x^2+y^2)=2a^5,$$
$$x+y=a.$$

2. Prove that for any two integers a and b the number

$$N=ab(a^2-b^2)(a^2+b^2)$$

is divisible by 30.

3. In a square $ABCD$ of area S the vertex A is joined by a straight line with the midpoint of the side BC, the vertex B with the midpoint of the side CD, the vertex C with the midpoint of the side DA, and the vertex D with the midpoint of the side AB. Find the area of that part of the square which contains the center of the square.

4. Prove that the midpoints of the straight line segments joining the center of the circle inscribed in a triangle with the centers of the three circles, each of which is tangent to a side of the triangle and to the extensions of the two remaining sides, lie on the circle circumscribed about this triangle.

5. Prove that if the real numbers p_1, p_2, q_1, q_2 satisfy the equality

$$(1) \qquad p_1p_2=2(q_1+q_2),$$

then at least one of the equations

$$(2) \quad x^2+p_1x+q_1=0, \quad x^2+p_2x+q_2=0$$

has real roots.

6. Solve the equation

$$x=a+\sqrt{a+\sqrt{x}}.$$

7. A rectangle $ABCD$ and a point M are given in three-dimensional space. Find the distance MD, having been given the distances MA, MB, MC.

8. Given an acute triangle ABC and a point M in the interior of one of its sides. Construct the triangle with the least perimeter such that one of its vertices is the point M and the two other vertices lie on the two remaining sides of the triangle.

9. Prove that a triangle can be constructed out of segments of lengths a, b, c if and only if

$$a^2b^2+b^2c^2+c^2a^2>\tfrac{1}{2}(a^4+b^4+c^4).$$

10. Prove that if $\alpha>0$, $\beta>0$, $\gamma>0$, and $\alpha+\beta+\gamma=\frac{1}{2}\pi$, then

$$\tan\alpha\cdot\tan\beta+\tan\beta\cdot\tan\gamma+\tan\gamma\cdot\tan\alpha=1.$$

11. In a triangle with sides a, b, c the segments m, n, p are drawn tangent to the circle inscribed in the triangle, so that their endpoints are on the sides of the triangle and they are parallel respectively to the sides a, b, c. Show that

$$\frac{m}{a}+\frac{n}{b}+\frac{p}{c}=1.$$

12. Prove that: a) the sum of the distances of an arbitrary point M in the interior or on the boundary of an equilateral triangle from the three sides of the triangle is constant, i.e., is independent of the position of the point M in the triangle; b) the sum of the distances of an arbitrary point in the interior or on the surface of a regular tetrahedron from the four faces of the tetrahedron is constant.

Results of the first round: A total of 7983 solutions of the problems were received, of which 6428 (i.e., 80.5 per cent) were graded at least "satisfactory" and 1555 (i.e., 19.5 per cent) were marked "unsatisfactory."

The second (regional) round took place March 2 and 3, 1956. In this, 326 students qualified from the first round, and 317 of these participated.

[8] The geometrical problems were to be solved by methods of synthetic Euclidean geometry only (and without the use of analytic geometry).

Problems of the second (regional) round

1. For which value of m is the polynomial $x^3+y^3+z^3+mxyz$ divisible by $x+y+z$?
2. Prove that if H is the point of intersection of the altitudes of a scalene triangle ABC, then the circles circumscribed about the triangles AHB, BHC, CHA, and ABC are congruent.
3. A horizontal circular disc of uniform density and weight Q kg is supported at three points A, B, C of the circumference of the disc, whereby $AC=BC$ and $\sphericalangle ABC=2\alpha$. What weight x kg must be placed on the disc at the second endpoint D of the diameter drawn from the point C so that the pressure of the disc on the support in C be equal to zero?
4. Prove that the equation $2x^2-215y^2=1$ has no solutions in whole numbers.
5. Prove that the numbers A, B, C defined by the formulas $A=\tan\beta\cdot\tan\gamma+5$, $B=\tan\gamma\cdot\tan\alpha+5$, $C=\tan\alpha\cdot\tan\beta+5$, where $\alpha>0$, $\beta>0$, $\gamma>0$, and $\alpha+\beta+\gamma=90°$, satisfy the inequality

$$\sqrt{A}+\sqrt{B}+\sqrt{C}\leq 4\sqrt{3}.$$

6. Suppose that in a tetrahedron $ABCD$ the segments joining the vertices of the tetrahedron with the centers of the circles inscribed in the opposite faces intersect at one point. Prove that

$$AB\cdot CD=AC\cdot BD=AD\cdot BC,$$

and show that the converse theorem is also valid.

The third round (finals) occurred on April 16 and 17, 1956. In this, 60 students qualified from the second round, and 59 of them participated.

Problems of the third round (finals)

1. Solve the following system of equations:

$$x^2y^2+x^2z^2=axyz,$$
$$y^2z^2+y^2x^2=bxyz,$$
$$z^2x^2+z^2y^2=cxyz.$$

2. Prove that if

$$(1)\qquad \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c},$$

and if n is an arbitrary odd natural number, then

$$(2)\qquad \frac{1}{a^n}+\frac{1}{b^n}+\frac{1}{c^n}=\frac{1}{a^n+b^n+c^n}.$$

3. On a straight line are given three distinct points M, D, H. Construct a right triangle for which M is the midpoint of the hypotenuse, D is the point of intersection of the bisector of the right angle with the hypotenuse, and H is the

TABLE 1

Appraisal of Solutions of the Problems of the Second Round

Grade[9]	Problems and numbers of solutions						
	1	2	3	4	5	6	Total
Very good (A)	57	46	26	18	21	14	182
Good (B)	36	40	12	19	11	16	134
Satisfactory (C)	32	29	14	32	20	23	150
Unsatisfactory (D or F)	117	169	174	206	144	86	896
No solutions	75	34	91	42	121	178	541

[9] The letter grades in parentheses are the closest American equivalents to the Polish grading system.

TABLE 2

APPRAISAL OF SOLUTIONS OF THE PROBLEMS OF THE THIRD ROUND

Grade	Problems and number of solutions						
	1	2	3	4	5	6	Total
Very good (A)	1	18	11	4	—	6	40
Good (B)	8	12	19	9	—	5	53
Satisfactory (C)	23	6	8	20	5	14	76
Unsatisfactory (D or F)	17	6	9	17	47	17	113
No solutions	10	17	12	9	7	17	72

foot of the altitude on the hypotenuse.

4. Prove that if the natural numbers a, b, c satisfy the equality

$$a^2+b^2=c^2,$$

then: 1) at least one of the numbers a or b is divisible by 3; 2) at least one of the numbers a or b is divisible by 4; 3) at least one of the numbers a, b, c is divisible by 5.

5. Prove that every polygon of perimeter $2a$ can be completely covered by a circular disc of diameter a.
6. Given a sphere of radius R and a plane α not having any points in common with the sphere. Let S be any point of the plane α. Consider the cone that has vertex S and is tangent to the sphere. If C is the center of the circle of tangency, find the locus of C as S moves freely in the plane α.

At the finish of the tournament fifteen winners were awarded the Diploma of the Mathematical Olympiad. Six other students were distinguished without diplomas. Twenty teachers whose students had received these honors were given honorable mention and received cash prizes.

These winning students received material prizes in addition to their diplomas: radios, watches, briefcases, fountain pens, cash prizes, books, and scholarships. The gifts came from the Ministry of Education, the Ministry of Higher Education, the Chancellors of Warsaw University and of the Warsaw Polytechnic Institute, the Executive Committee of the Olympiad, and the Central Office of Professional Training. These awards were conferred at a solemn ceremony in the Conference Hall of the Ministry of Education in Warsaw on June 16, 1956.

A report of the whole tournament, including the problems, their solutions and analyses (by Professor Straszewicz), lists of the participants of the third round, the winners, and their teachers, was published by the Ministry of Education and then distributed to all the secondary general-education and semiprofessional schools.

Not fewer than 80 per cent of those graduating from the [university] faculties of Philology, History, Geography, and Biology; and not fewer than 60 per cent of the graduates of the Physics, Mathematics, and Chemistry faculties shall be directed to teach in secondary schools.—*Alexander G. Korol*, Soviet Education for Science and Technology, *p.* 272.

Coordination and possible curricular changes[1]

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How can the colleges work more effectively with the high schools? What is the best way to get continuity in high school courses and the necessary mathematical competence?

A COMMITTEE to coordinate mathematics courses in the high schools of our region with desired preparation for Tech courses has been operating throughout the school years of 1955–1957. We have developed some ideas that seem to be worthwhile and so I am offering them to you of other parts of the country.

At luncheon meetings we have been able to tell the high school mathematics teachers about many things. Among them are the demand of industry for mathematics majors; the equality of salaries of all the science graduates; the many new fields, other than teaching, for mathematics majors. We have set up a lecture bureau for those clubs interested in mathematics, even "Minute Men" to talk to small groups or individuals about the possibilities for mathematics majors. We have set aside a scholarship for a student who wishes to major in mathematics, said student to be chosen from those taking an examination each spring.

Also we have been able to tell the teachers of bright students about the topics most necessary for giving said students the most useful foundation for a technical college course. For instance: the definitions of the trigonometric functions for the general angle, analytic trigonometry, solutions of equations, graphs, and inverse trigonometric functions. We suggest that these teachers cut short the solutions of triangles and uses of tables and the right-triangle treatments. Also we have pointed out that a very few plane geometry facts are really essential. It is sufficient to teach the ideas of similar triangles, parallel lines cut by transversals, circles and inscribed angles, mensuration formulas, the Pythagorean theorem, and that the sum of two sides of a triangle is greater than the third side. All other ideas are of value mostly for development of logical reasoning and proper orderly presentation. Much time may be saved in plane geometry courses without loss for the student.

Furthermore, solid geometry may be dropped as a semester course. The three-dimensional concept and the ability to draw and see solids with the proper perspective are all that the college student needs. These facts, if accepted, give much more time for algebra and the essential topics in trigonometry.

Even much time-killing algebraic work should not be done. Such items include the idea that improper fractions have some kind of stigma, that they must be changed to a mixed number or inaccurately expressed as a decimal—which is utterly absurd. Then, too, the square roots of arithmetical numbers should be left in

[1] From speeches presented to the Allegheny Mountain Section of the Mathematics Association of America, April 28, 1956, and May 4, 1957.

that form. The rationalization of denominators is another anachronism. Often the numerator needs to be rationalized, but never the denominator.

If teachers used their time on essential items instead of such ones as are mentioned above, they would obtain much better results. Teachers should use this extra time on statement problems, graphs, solutions of equations, partial fractions, and the binomial theorem.

Also we are able to point out to these teachers that their jobs are important; that their efforts start bright students upon the roads for very useful and satisfactory lives; that they are lucky in having a *subject to teach* in contrast to much that is done in high schools today.

Above all, we have been able to point out that no profession gives one the satisfaction that teaching does. Each student who has become a top man in his field and returns to my office to say, "Doc, you found me when I was lost and put my feet on the right road," gives me more than enough pay for all my efforts. Can any other profession meet such returns? I would not trade professions with any man.

Our efforts are met more than halfway by the teachers who came to our luncheons. We hope this can become a nationwide move. We really need a crusade as of now.

This same committee at the Carnegie Institute of Technology has arrived at another worthwhile idea, which—evolved by teachers of mathematics in our region and a member of the Pennsylvania Department of Public Instruction—is a list of the topics which seem desirable for prospective college students.

In building such a list, we started with one proposed by the University of Illinois in 1951. After many committee hours and changes we offer the one given below.

It is evident that the list is a minimal one in which the topics are of unequal importance. Also that the use of it will give the individual teacher the responsibility for the time devoted to each item.

However, it includes the essentials for a successful continuation of mathematics in college and definitely does not suggest that the college courses in analytic geometry and calculus be attempted in high schools. Our committee and staff feel very strongly that those subjects should be left to the colleges and that the list contains enough for four years of high school mathematics.

We hope the following will be a help to high school teachers and will do something toward better standards in the high schools of our country.

DESIRABLE TOPICS
FOR PROSPECTIVE COLLEGE STUDENTS

1. Fundamental operations (addition, subtraction, multiplication, and division) with integers, common fractions, decimals, and mixed numbers.
2. Concepts of percentage, per cent of increase and decrease.
3. Concepts of ratio and proportion.
4. Concept of measurement and standard units.
5. Expression of a physical magnitude: number and units.
6. Conversion of units in physical magnitudes.
7. Problems involving lengths, areas, angles, etc.
8. Concepts of precision of measurement, significant digits, and rounding.
9. Concepts of variables and constants.
10. Treatments of parentheses, brackets, etc.
11. Concept of directed or signed numbers.
12. Fundamental operations with signed numbers.
13. Definition and fundamental operations with polynomials.
14. Fundamental operations with algebraic fractions.
15. Special products: i.e., $a(a+b)$, $(a+b)^2$, $(a+b)(a-b)$, etc.
16. Factoring.
17. Laws of exponents, negative, fractional, and operations with radicals.
18. Rectangular coordinates, *thoroughly*.
19. Solutions of linear equations.
20. Solutions of systems of linear equations, including solutions using determinants.
21. Concept of variation; absolute value.
22. Concepts of a function, functional notation, representation as graphs from tables of related values.
23. Properties of a linear function—i.e., graph, standard form of a linear equation, slope, and intercepts.
24. Solution of a quadratic equation by factoring, by completing the square, and by formula.
25. Fundamental operations with complex numbers.

26. Standard form of the quadratic equation, the nature of its roots; sum and product of roots.
27. Solution of a system consisting of a linear and a quadratic equation.
28. Solution of simple pairs of quadratic equations.
29. Solutions of verbal problems by algebraic methods.
30. Solution of equations in which the unknown occurs under a radical sign.
31. Binomial theorem with positive integral exponents.
32. Computation by use of logarithms; interpolation.
33. Remainder and factor theorems, synthetic division.
34. Rational roots of $f(x)=0$, where $f(x)$ is a polynomial in x of degree higher than 2.
35. Such conventions as $a^{1/2}=\sqrt{a}$, meaning a positive number or zero.
36. Approximation of the irrational roots of equations.
37. Sequences and series; applications of arithmetic and geometric series; infinite geometric series.
38. Concept of equality, including the postulates of equality.
39. Inequalities.
40. Concept of a definition, postulate, theorem and its converse.
41. Deductive proof.
42. Inductive reasoning; its use in science; the difference between inductive reasoning and proof.
43. Use of the compass and straightedge for constructions.
44. Concept of plane angle; dihedral angle.
45. Polygons: triangle, rectangle, parallelogram, trapezoid.
46. Circles: constructions of circle tangent to a line and to another circle.
47. Angle inscribed in an arc of a circle.
48. Mensuration of plane figures.
49. Geometric relations among triangles; congruences and similarity.
50. Concept of symmetry as to a point; line, plane.
51. Concept of locus.
52. Parallel and perpendicular lines.
53. Pythagorean theorem.
54. Projection.
55. Perspective representation of three dimensions.
56. Parallel and perpendicular planes; line and plane.
57. Polyhedrons: cubes, prisms, pyramids.
58. Cylinders, cones, spheres.
59. Definitions of trigonometric functions of a general angle, using rectangular coordinates.
60. Solutions of a few typical right triangles with several applications.
61. $\sin(90°-A)=\cos A$ and other reduction relations.
62. Values of the functions of special angles, including quadrantal angles.
63. Use of natural and logarithmic tables involving trigonometric functions.
64. Fundamental trigonometric identities of an angle.
65. Proofs of other less obvious identities by use of the fundamental identities.
66. Law of sines; law of cosines.
67. Solutions of several oblique triangles, using both natural and logarithmic tables.
68. Radian measure of angles.
69. Graphs of trigonometric functions.
70. Principal values of inverse trigonometric functions, particularly arc sin, arc tan.
71. Solution of trigonometric equations.
72. Double-angle identities.
73. Half-angle identities.

> Once the theory (logarithms) is carried to this point, the logarithmic tables are put into the hands of the pupil and he must learn to use them in practical calculation. There may still be some schools—in my school days this was the rule—where little or nothing is said as to how these tables are made. That was despicable utilitarianism which is scornful of every higher principle of instruction, and which we must surely and severely condemn.—*Felix Klein*, Elementary Mathematics from an Advanced Standpoint: Arithmetic, Algebra, Analysis (1908).

Research in teaching high school mathematics

KENNETH E. BROWN, *U. S. Office of Education, Washington, D. C.*

Have educators been asking the right questions in their research studies? Is it possible that more studies orientated toward fundamental psychological processes would prove profitable?

WHAT DO RECENT EDUCATION STUDIES show about the most effective ways of teaching high school mathematics? A survey[1] of the research in mathematics education for the calendar years 1955 and 1956, conducted by the U. S. Office of Education in cooperation with the Research Committee of the National Council of Teachers of Mathematics, showed considerable experimentation with the content of high school mathematics.

The number of pupils involved in the different experiments ranged from many pupils in several schools to only a few pupils in a single class. In one state a group of 36 teachers developed a sequence of topics for the entire four-year high school mathematics program. The content was from algebra, geometry, and trigonometry, with applications to social problems. In another state, teachers from several schools are trying out not only new content but a different emphasis on some of the traditional topics. In contrast to these experiments embracing hundreds of pupils was an experiment with one elementary algebra class taught by several staff members of a large city college.

[1] Kenneth E. Brown, *Analysis of Research in the Teaching of Mathematics 1955–1956*, Bulletin 1958, No. 4 (Washington: U. S. Government Printing Office, 1958).

ANALYSIS OF TEXTBOOKS

There was experimentation in high school general mathematics and analysis of currently used textbooks. Studies showed considerable variation in the content of general mathematics textbooks; however, they usually included topics from elementary algebra, informal geometry, and arithmetic, as well as applications to the solution of concrete problems. In fact, some of the general mathematics textbooks were mostly arithmetic. Many of the pupils who take general mathematics do not go on to college. For this reason, some educators have looked to the mathematics used by semiskilled and unskilled workers as a key to the content of general mathematics. One mathematics education investigator concluded that seventh-grade mathematics would fill the needs of most workers.

The principle of building a curriculum on the present use of mathematics can be questioned: (1) Is not such a procedure a status-quo approach without provision for advancement? (2) Would not industry require a greater use of mathematics if the workers had greater ability in the subject? (3) Is it not likely that when the present pupils are adults, a greater knowledge of mathematics will be demanded of semiskilled workers than is now the case?

It would seem that the space age is going to require more education of each person, especially in the field of mathematics, in order to understand daily events and be an intelligent citizen.

Geometry

Although there was experimentation in the teaching of geometry, no special techniques were reported that have promise of changing greatly the present practice. Teaching for meaning through discovery and broader generalizations were paramount in some of the studies. In one experimentation, two groups of pupils were matched as to mathematics achievement and I.Q. One group was taught plane geometry by the traditional method, and the other group used specially prepared material designed to lead to the discovery of relationships and broad generalizations. However, the investigator concluded that the two methods were equally effective.

Different results were reported from similar experiments in the teaching of solid geometry. One extensive study concluded that little, if any, improvement in space conceptions resulted from the study of solid geometry. Another experiment showed significant gains when the analytic approach was used.

Algebra

As in geometry, no research showed one best method of teaching algebra. A study of elementary algebra textbooks, for example, revealed nine different methods of teaching signed numbers. In an experiment with 55 pupils the discovery method was used in teaching a unit of directed numbers. The achievement of the experimental group was about the same as that of the control group.

In another more extensive experiment involving ten algebra classes, the method of exploration and discovery was used with the experimental group in contrast to drill with the control group. The program was evaluated by the Douglas Survey Test and a special test developed in harmony with objectives of the experimental group. The Douglas Survey Test showed no significant difference in achievement between the two groups. The special test results favored the experimental group; however, both tests showed that a large percentage of pupils made little achievement. Much has been written on the need for using the method of exploration and discovery in teaching algebra, but little objective evidence was reported to prove that this method is being used. Since exploration and discovery does not lend itself readily to the traditional lecture and recitation method, perhaps this is one reason why it is not being used. A random sample of 131 teachers in one state indicated that they preferred the traditional lecture-recitation method above all other methods. Nearly one-half of the teachers stated that student committee work is never or seldom valuable.

Homogeneous grouping

In addition to studies on methods of teaching individual subjects, there were studies on organizational procedures. For example, one study dealt with the effect of homogeneous grouping upon pupil achievement. Evidence was not conclusive, but data seem to indicate that success depends on the area of expected achievement. In one experiment the homogeneously grouped pupils made greater achievement in fundamental mathematics skills but less achievement in problem solving than those not so grouped. One investigator reported that approximately two-thirds of the schools in his study used some kind of homogeneous grouping. In some cases the grouping was limited to a two-track program in ninth-grade mathematics. Special programs were usually in schools with enrollments above 1000. Where homogeneous grouping is received favorably, usually a combination of criteria has been used to determine the groups. The criteria are ratings on I.Q. test and mathematics achievement test, previous marks in mathematics, and

student-interest ratings. The studies reported little or no difference in methods of instruction for the different sections of homogeneously grouped pupils. Methods for teaching the superior pupil were similar to those for the slow pupil. One survey reported that for 95 per cent of the schools the chief means of providing for the gifted was through enrichment.

Experiments were reported also on the success of homogeneous grouping within the classroom. These experiments showed no significant difference in achievement of high school pupils taught in small groups within a class as compared to those taught in one group in a class.

In this period of competent mathematics-teacher shortage, many physical devices have been suggested for teaching the subject. These seem to make only a small contribution to more effective teaching of mathematics. Like the chalk board, the device itself seldom teaches but when it is used by skilled teachers it aids instruction.

Summary

Several of the studies on the teaching of high school mathematics emphasized understanding concepts. The teaching methods for the experimental groups, no matter whether they involved physical aids or homogeneous grouping, seemed beneficial to some extent when used by a skilled or specially trained teacher. No doubt research by frontier thinkers in mathematics education is most valuable, but if implementation is to follow, research will need to determine the specific changes possible with the present teachers. For example, how much modern mathematics and which modern mathematics can present teachers incorporate in their instruction? How much statistics can be taught by the present teacher untrained in the subject? How much change can be made in high school course content and still give security not only to the teachers but to the parents? These and related questions remain unanswered.

Even a casual review of the studies would reveal certain student studies of inferior quality that contribute little to the teaching of mathematics. Likewise, some faculty member studies differed materially in quality and degree of contribution to the teaching of mathematics. Little of the research was the result of a team approach to the solution of a critical problem in this area on any of the levels—elementary school, high school, or college.

The questionnaire used to gather the research asked the respondent to indicate one or two crucial problems in the teaching of mathematics. These problems, reported in the bulletin, indicate a great variety both as to significance and type, but nevertheless may not be the really crucial problems in mathematics education. The studies reflect a need for State groups of teachers or National committees to identify the really crucial key problems so that direction may be given to research in the field.

The studies reflect another need—greater coordination of effort in attacking identified problems. Many are too large to be solved by a single individual; teamwork will be required. The entire mathematics staff of a school system may profitably concentrate its research efforts on a single problem.

Finally, the studies reflect a third need—to publish research results and distribute them widely. Some of the investigations, reported in the present U. S. Office of Education Bulletin, resulted in compilations of teaching material or detailed experiments which were never published. Such unpublished research has little impact on classroom practice.

Improvements in mathematics teaching will depend upon the extent to which we identify the crucial problems, coordinate our efforts to solve them, and make the results known to all classroom teachers.

What is happening to the potential scientist or engineer?

GLEN D. VANNATTA, *Broad Ripple High School, Indianapolis, Indiana.*

Do our best high school students go to college? What role does a teacher play in determining whether capable students go to college? This article supplies some possible answers.

THE SCIENTIFIC MANPOWER SHORTAGE has been in the spotlight for several years and the problems and proposals pertinent to it have been the central theme of conventions, committee endeavors, and individual effort. Research and discussions have focused on high school science and mathematics programs, for it is in the high school that most of the decisions are made that determine higher educational pursuits and vocational choices.

The study reported here was directed toward discovering from the potential scientist himself what the factors are that characterize and contribute to the high school background and higher educational objectives of superior mathematics students. Questions which bear on the problem are:

1. Are potential scientists, engineers, and mathematicians who are recent high school graduates continuing their formal education in college?
2. What factors influence the decision to go to college for these groups?
3. What vocational choices in college are being made by students with high scientific potential?
4. What factors influence these vocational choices?
5. What factors develop interest in mathematics and science?
6. What phases of scientific work appeal to superior high school graduates?
7. What can the high school do to increase its effectiveness so that greater numbers of superior graduates are encouraged to go to college?

When the problem was chosen it was desired that a group of recent high school graduates with high scientific potential be selected to become participants in the study. Such a group was found in those who qualified for the final competitive examinations in Comprehensive Mathematics, a section of the Indiana State High School Achievement Program. The examination is open to Indiana high school students who have pursued mathematics through three or more years, and the numbers are limited by a regional examination to about 125 per year. The span of years chosen was 1952, 1953, 1954, and 1955, since these were the years when the scientific manpower shortage rose to crisis level.

Once the group of 406 proven superior high school mathematics students had been selected, a questionnaire was formulated to ascertain information about them that had a bearing on the problem. The questionnaire was prepared in rough form and submitted to teachers, professors, and others who had an interest in the problem. Also a pilot study was made with 15 high school students who had participated in the Comprehensive Mathematics Program but who had not qualified for the

final examination. From these preliminary reviews and tryouts the final form of the questionnaire was prepared, printed, and mailed to the 406 participants. A total of 327 or 80.54 per cent returned usable responses. The data were coded and transferred to IBM punch-cards for facile handling.

The participants in this study were selected because there was reason to believe they possessed high scientific potential. The findings confirmed this belief to a very high degree in that 76.58 per cent of all respondents had made vocational choices that were classified as being in the science areas.

The first part of the findings considers the nature and background of the participating group. Of 327 respondents, 271 were males and 56 were females. The median age of the respondent group as of September, 1955, was 19.0 years and the range was from 16 to 22 years of age. Nearly one-fourth of the respondent group ranked first in their respective high school graduating classes and the median rank in class was 3.1. The median size of graduating class was 114.

A very high per cent of the respondents had attended or were attending college; 96.64 per cent were reported in this group. The state universities claimed nearly half the respondent group, with Purdue University enrolling 31.5 per cent. Colleges outside the state of Indiana enrolled 21.41 per cent.

Interest in mathematics and science was very high. This is indicated by the data which showed 98.47 per cent had rated interest in mathematics as very high or high; 82.56 per cent had ranked science in these categories.

Respondents were generally aware of the problem of scientific manpower shortages while in high school; the awareness had increased over the span of the years studied. Teachers and news publications shared major credit for bringing about the considerable awareness of the problem.

Over one-third of the respondents had taken full-time or part-time jobs, but very few of the jobs were in the science areas. There were 21 respondents who had married and nine who had been or were in military service.

Over three-fourths, 78.9 per cent, of the respondents had received scholarships. A surprising fact was that a smaller per cent had received scholarships during the last two years of the study than during the first two years. About 15 per cent of the scholarships were valued at over $1000.

The reasons for not going to college or dropping out were dominated by marriage or financial problems. The mean age of respondents when the decision to go to college was made was the fourteenth year.

The second part of the findings pertains to the choices and opinions of the participating group. The mathematics teacher was ranked as having by far the greatest influence on stimulating interest in mathematics and/or science. A total of 43.73 per cent considered the mathematics teacher the most important factor and 75.84 per cent indicated the teacher as first, second, or third in importance. Parents were ranked second in this regard, with a combined frequency amounting to 31.8 per cent. Books, newspapers, and laboratory work in school also ranked high in developing scientific interest.

The vocational choices of the respondent group were largely in the science areas. The most popular choice was engineering, which claimed 36.08 per cent. Mathematics or science teaching was chosen by 11.71 per cent of those who had gone to college. Medicine, physics, chemistry, and mathematics accounted for most of the remaining science vocational choices. When vocational choices were examined on a year-by-year basis, it was revealed that the per cent of respondents choosing science careers increased from 69.7 per cent in 1952 to 83.12 per cent in 1955. Engineering per cents increased more than other science areas.

A comparison of males and females revealed that less than half of the female re-

spondents chose science vocations, but nearly 80 per cent of the males made this choice. Females chose teaching much more frequently than did males. A much higher per cent of the females had not gone to college, had dropped out, and had married.

Engineering and mathematics were the first- and second-ranked college major subjects by a wide margin. Physics and chemistry were third- and fourth-ranked respectively.

There were 49 college respondents who had changed their major subjects while in college. Most of the changes were within the science fields, but when changes occurred involving science and nonscience areas, the changes from science areas to other areas predominated by nearly four to one. Reasons given for changes of major were largely due to the recognition of greater personal suitability to the new major by the respondents involved.

The dominant factors which influenced the decision to select a college major were the high school teacher, the father, and success in school work. Special talent and the promise of good income were important as secondary factors.

The decision to go to college was influenced by the parents, the teacher, the personal desire for more learning, and success in school work. Higher income level for college-trained people was an important secondary factor. Athletics, prestige, and social life were rated as not having great importance to this group in making the decision to attend college.

The phases of scientific work that appealed most to the respondent group were the mental stimulation, the challenge of the unknown, and the opportunity for creativity. Financial rewards were of secondary interest, but mechanical aspects held little attraction.

Respondents felt that the most important things the high school could do to encourage more able students to continue their education in college were to vitalize teaching and provide more and better individual counseling. Publicity regarding scholarships was considered very important, as was a good preparation in the fundamentals of secondary education.

Great interest in the problem was elicited and more than half the respondents expressed themselves freely on various points that should be considered in attempts to solve the problem.

A number of significant conclusions were derived from the study. They are summarized as follows:

1. Participants in competitive mathematics and science programs are, for the most part, potential scientists, engineers, or mathematicians. Their interests and abilities in mathematics and science are high. The competitive programs are moderate stimuli to further study in the area.
2. Superior mathematics students are generally aware of the current shortage of scientists, engineers, and mathematicians. Teachers and news publications are largely responsible for bringing about this awareness.
3. A very high per cent of superior high school mathematics students attend college. The decision to attend college is made during the fourteenth year on the average for this group. Marriage and financial reasons are the greatest factors that cause students to drop out of college or prevent their attending. The state universities attract a high per cent of students with scientific potential; about three-fourths attend colleges within the state.
4. More than 50 per cent of the superior mathematics graduates take part-time or full-time jobs while in college. The jobs are usually not in the science fields.
5. A high per cent receive financial aid in the form of scholarships but the amount of the scholarship is often limited.
6. Males rank somewhat higher than females in competitive mathematics programs and choose science-related careers more often than do females.

7. The high school teacher has paramount influence on the development of interest among superior students in the fields of science and mathematics. The high school teacher also has greatest influence on the choice of college major made by superior mathematics students under his guidance.
8. Parents have great influence on the higher educational plans of their children, but are second in importance to the teacher except in the actual decision to go to college.
9. Personal desire for learning and success in school work has considerable influence on the higher educational choices made by superior students.
10. School administrators and advisors as well as adult friends and relatives have limited influence on the educational choices of such students. Friends have moderate influence on these choices.
11. The promise of good income has considerable influence on the career choices of college students.
12. Between 1952 and 1955 there has been a significant increase in the per cent of potential scientist and engineer personnel who, after high school, have chosen science areas for a career. Engineering accounts for one-half the number of cases in this increase.
13. The most appealing facets of scientific work are the mental stimulation, the challenge of the unknown, and the creative opportunities offered.
14. Mathematically superior high school graduates are eager to cooperate in working out answers to the problem of scientific manpower shortages. They have a great interest in the problem and the possible solutions of it.
15. The attack on the problem of scientific manpower shortages focuses on the secondary school teachers of science and mathematics. Materials and equipment make a substantial contribution in encouraging students with scientific potential to make science-related vocational choices while in college. Services and activities are of even greater importance. Parents have considerable influence, as do friends. But the problem will not be solved by providing more and better services, more and better equipment, more effective publicity, and more and larger scholarships. The heart of the solution lies in the development of a corps of well-trained, enthusiastic, stimulating, and diligent teachers of science and mathematics. An environment that produces high morale and adequate financial rewards will be necessary to attract and hold such teachers in the profession.

> We grant that any thoughtful American reader encountering statistical data on Soviet education may instinctively compare them with the American counterparts and be tempted to use them as a gauge of American performance. But such comparisons, however valid in detail they may be, miss the very essence of the problem and obscure the fundamental noncomparability: the difference in the goals, the philosophy, the operating rationale, and the functions of education in the two systems and the magnitude of the tasks which the two systems respectively must fullfill to achieve their objectives. Only in such a frame of reference can the Soviet educational performance be seen in its proper scale as we have endeavored to show it in this survey.—*Alexander G. Korol,* Soviet Education for Science and Technology, *p.* 408.

On the psychiatrist's couch

ROBERT T. BLACKBURN, *Shimer College, Mount Carroll, Illinois.*

Words, words, words!

AN OBVIOUSLY GIFTED eighteen-year-old boy had caused his parents some concern with what they called his "childlike" imagination. Rather mild persuasion resulted in a session with a psychiatrist. The following is a record of a portion of their conversation.

"And I suppose you have a lucky number?"

"Natch."

"Have you ever lost a bet on it?"

"Nope."

"Never?"

"Never."

"Then I suppose that you've won a good deal betting on it?"

"Nope—not a cent."

"Oh, I see. You don't gamble."

"What makes you say that? Of course I do."

"I beg your pardon! You've never lost on it and yet you've never won. That hardly seems natural, does it?"

"I didn't say it was natural. In fact, it is not."

"That's odd. I . . . "

"It's not odd."

"Well, even if . . . "

"It's not even either."

"Well—perhaps I used the wrong word. I wanted to question if it was rational the way . . . "

"Who said the number was rational? It's not."

"Uh? . . . but this lucky number of yours—it seems to be an integral part of . . . "

"No."

"But you as much as said that it was important to you and . . . "

"Sure it is."

"But not an integ—?"

"Right."

"Uh? . . . Maybe I'm a square, but . . . "

"It's not a square."

"Hmmm . . . I said maybe *I* was a square. I didn't say anything about your lucky number having anything to do with squares. I was only . . . "

"But it is involved with squares. It cuts them all."

"A little more slowly, please. We're getting off the track. Let's see . . . are you sure this lucky number of yours is not imaginary and a figm—?"

"Absolutely not!!"

"Excuse me. I didn't mean to offend you. I just . . . "

"You didn't. But what you intimated, while not absurd, is—well, I've never heard of anyone with an imaginary lucky number. Have you?"

"No—but you said before it wasn't natural or rational and it wasn't an integral part and I just thought it might be possible that . . . "

"Very clever—possible indeed, but not the case. My number is real."

"Ah . . . this is getting complex. I suggest . . . "

"Oh—it's not complex. But *touché*. I hadn't thought of that possibility. A complex lucky number . . . hmmm . . . very exciting. I'm sorry . . . you were saying?"

"Well . . . uh . . . let's see. Your lucky number is certainly unique."

"Every number is, isn't it?"

"No . . . yes . . . no, I meant you are unique in having this number as your lucky number."

"How do you know that?"

"I don't. What I meant was . . . . What I . . . . Say, what is your lucky number?"

"Isn't it obvious?"

"Hardly."

"But I've told you all you need to know. There is only one number it could be."

"Young man, I've had a busy day. This is not a game."

"Oh?"

"No. Now I'll ask you once more. What is your lucky number?"

"The square root of 2, obviously. What's yours?"

While this last exchange had been transpiring, the nurse had quietly entered and placed coffee and two pieces of cherry pie in front of the psychiatrist, his usual mid-afternoon snack. Obviously slightly dazed, and from habit, the psychiatrist picked one piece and extended it towards the unobserving youth.

"Uuuuuuuh? . . . Pie?"

"Pi!! You're no square. . . . Crazy, man . . . "

What America needs today, then, is " . . . a schooling better aware of its aims."—In a real sense, the future of our civilization depends on the direction education takes, not just in the distant future, but in the days immediately ahead.—*President's Commission on Higher Education.*

Ode to Q.E.D.

Hail to thee
    O Q.E.D.—
At the end of each proof you are found.
While mathematics to me
    Was a turbulent sea,
You represented dry ground.

My inadequate brain
    Strove to conjure again
The steps that the author deleted,
As you played double role
    (Both inspirer and goal)—
A Greek phrase for "mission completed."

Quite early in youth
    I discovered the truth—
I've no mathematical leaning.
But years of exposure
    Have brought me composure,
And you, Q.E.D., a new meaning.

You're the first thing I see
    That's familiar to me,
Though the last thing that catches my eye;
Then, hail to thee!
    O Q.E.D.,
Greek for "God only knows why!"

—*Luella Ellington, University of Maine, Orono, Maine*

A method for finding the extent of $y = \sqrt[n]{f(x)/g(x)}$

LOUIS L. PENNISI, *University of Illinois, Navy Pier, Chicago, Illinois.*

An interesting technique for determining the range of a function. It can be used in advanced high school classes.

THE PURPOSE of this classroom note is to give a method for finding the range of the real values of x for the function

$$(1) \qquad y = \sqrt[n]{f(x)/g(x)},$$

where

$$(2) \qquad f(x) = \prod_{i=1}^{k} (x-a_i)^{\lambda_i}$$

and

$$g(x) = \prod_{j=1}^{l} (x-b_j)^{\beta_j},$$

a_i, $b_j (a_i \neq b_j)$ are real numbers, λ_i, β_j are positive integers, and n is an even integer.

Observe that $f(x)/g(x) > 0$ if and only if $f(x)g(x) > 0$. Let

$$(3) \quad h(x) = f(x)g(x) = \prod_{i=1}^{k} (x-a_i)^{\lambda_i} \cdot \prod_{j=1}^{l} (x-b_j)^{\beta_j}.$$

Note that at $x = a_i$ and $x = b_j$ the graph of $h(x)$ is either tangent or crosses the X-axis for all λ_i and β_j. Thus the graph of $h(x)$ may be quickly sketched. (See figure below.)

Hence, the desired range is given by those real values of x for which $h(x) > 0$, and also for $x = a_i (i = 1, \cdots, k)$.

EXAMPLE: Find the extent of

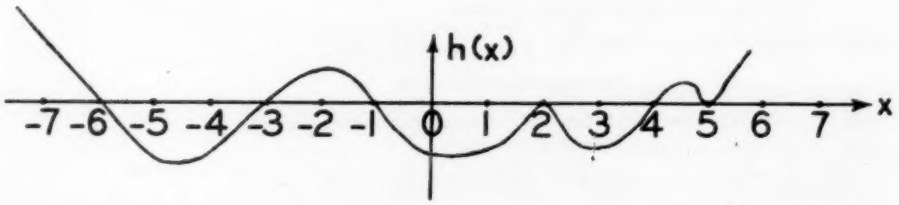
$$y = \sqrt[4]{\frac{(x+3)(x-4)^2(x-5)^2}{(x+1)(x+6)^2(x-2)^2}}.$$

$h(x) = f(x)g(x)$

$= (x+3)(x-4)^2(x-5)^2(x+1)(x+6)^2(x-2)^2.$

Hence, the desired range is given by:

$$x \geqq 4, \qquad -3 \leqq x < -1, \qquad x < -6.$$



Edited by Howard Eves, University of Maine, Orono, Maine

Reuter, Gauss, and Göttingen

by Laura Guggenbuhl, Hunter College, New York, New York

At the end it took just a few dots and dashes and a whisper to prove that it could be done. The dots and dashes spelled out a singularly uninspired and unimaginative sentence, but it was enough to prove that a message could be sent over a wire. It is no great wonder that the good burghers of Göttingen were fearful and suspicious of the mysterious black wires which Gauss had strung above the rooftops of the town from his astronomical observatory to their beloved Johanniskirche. And it is no great wonder that the successful outcome of Gauss's experiments with electromagnetism caused a tumult and excitement of epochal proportions in this little university town.

In 1816, when Gauss was thirty-nine years old, he took up residence as the first director of the newly-built astronomical observatory at Göttingen. Today, from the outside, a stranger could easily mistake this building for a modern structure, for it has the ageless beauty and quiet charm of simple design and classic lines. It is surrounded by well-tended and spacious lawns. One sunny wing of the building, with private entrances and gardens, was reserved as a residence for Gauss and his large family.

Gauss lived and worked at the observatory until his death, thirty-nine years later. He did not like to travel, and from 1828 until the day of his death in 1855, he had spent only one night away from the observatory. On his last overnight trip away from Göttingen in 1828, he met Wilhelm Weber at a scientific meeting in Berlin. Shortly after this meeting, Weber became Professor of Physics at the University of Göttingen and a close collaborator with Gauss in his research in the field of electromagnetism. In the spring of 1833, their experiments were crowned with success when they sent a message over an electric telegraph wire.

Two parallel strands of wire had been strung from the observatory to the physics laboratory of the university, about a mile and a half away. The wires ran above the rooftops of the town and were attached to one of the towers of Göttingen's Johanniskirche and then lowered to the physics laboratory in the next block. The first message over the wire was the statement, "Michelmann is coming." And, according to legend, Michelmann arrived before the message was decoded.

It was not long before the wires became the subject of heated debate in Göttingen. The town's inhabitants were afraid that the wires might attract lightning and that the towers of the Johanniskirche might be damaged. There was a lengthy, well-documented, and minutely-recorded exchange of correspondence, and the subject was discussed at length in legislative councils. After sufficiently reassuring arguments had been made, the wires were allowed to remain in place. They were finally destroyed by lightning (and not replaced) in 1845.

As is so often the case in scientific discoveries, Gauss seemed to have no interest

whatsoever in the commercial exploitation of his invention. After he had proved that a message could be sent over a wire, he turned his attention to other interests. The commercial and financial development of the invention was left to others—in particular to a young boy who had been sent to Göttingen to live with an uncle after the death of his father.

The youth's name was Israel Beer Josephat, but in later years he was to become famous as Paul Julius Reuter. He was the third son in a family of four children, whose father was a rabbi in the nearby town of Cassel. After the death of his father, when he was about sixteen years old, he went to Göttingen to live with an uncle. Shortly after his arrival in Göttingen he began to work as a messenger boy in his uncle's bank on Prinzen Strasse, a next-door neighbor to the physics laboratory of the university. A warm friendship developed between Gauss, who was then about sixty years old, and young Josephat. One can only speculate about how deep an impression was made upon the lonely and bereaved youth by the town-stirring inventions of the scientist.

Presently the young man tired of his work at the bank, and he moved on to Berlin and then later to Paris. Berlin records show that he became a Christian and that he changed his name to Paul Julius Reuter. In Berlin he was a partner in a publishing house, and in Paris he was a translator in the Havas News Agency.

In the meantime, in the years between 1833 and 1850, the Gauss-Weber scientific invention had become an important commercial enterprise, and telegraph wires had been strung connecting one large city with another in various parts of Europe. One such line went from Berlin to Aachen, and another connected Paris with Brussels. At first these wires were not open to the public, and even in later years there were many limitations upon their use. Reuter had been a deeply impressionable eyewitness at the birth of this discovery, and it was surely just a matter of time before it would become an all-absorbing part of his life. In Berlin there was a well-established news agency known as "Wolff's Bureau," and the Havas News Agency had its headquarters in Paris. Reuter rented a little office at Aachen and set up a business of his own. Perhaps it was the influence of the Göttingen professors which led him to call his business an "Institute" rather than an "Agency" or a "Bureau." Carrier pigeons were used to close the gap between Brussels and Aachen. As time went on, more and more telegraph wires were strung, and it was not long before a wire was strung from Brussels to Aachen. The pigeon post from Aachen to Brussels became obsolete, and the future for Reuter's little business at Aachen was no longer as promising as it had been a few years earlier. After being advised by friends to go to Great Britain, Reuter sold his business at Aachen and moved to London. In 1851, with a staff of three, consisting of his wife, a young office boy, and himself, he laid the foundations in London for the world-renowned organization which is now known simply as "Reuters."

In London, Reuter's success was spectacular and phenomenal. He had become a naturalized British subject; he was made a baron by the Duke of Saxe-Coburg and Gotha; and, by a special grant of Queen Victoria, he and his heirs were authorized to have the privileges of this rank in England. During the latter years of his life, his duties at the news agency were gradually turned over to his eldest son, and Reuter became interested in banking once again. He died in Nice in 1899, forty-four years after the death of Gauss.

The blessings of the little university town had spread out far and wide—from the tiny Bruder Grimm Allee to a home in Nice on the French Riviera, from a small bank on the Prinzen Strasse, surrounded by a medieval convent and timbered houses and next door to a physics laboratory so old that it was soon to be torn down, to far-flung interests in the Bank of

Persia. Whether Reuter ever went back to Göttingen, one cannot say.

The history of Reuters reached heights and touched depths during the years when peoples were torn asunder by war. The agency had been founded a few years before the American Civil War. It prospered and grew by leaps and bounds as the result of dramatic achievements of reporting during the years 1861–1865. In the war years of 1915 and 1916, the personal history of the Reuter family was marked by heavy tragedy. The eldest son of the founder died, a suicide, in 1915. The founder's grandson, Baron Hubert de Reuter, who had no interest whatsoever either in banking or in the family news-gathering empire, had selected as his profession the vocation of schoolmaster. In 1916, Baron de Reuter, the last male descendant in direct line of the founder, was killed in action while serving as a private during the fighting on the Somme.

Now, at the beginning of the second century of its existence, Reuters is no longer a family enterprise. In fact, there is now no direct personal link between the founder and the news service. And thus, the name which Israel Beer Josephat had selected for himself from the world at large represents, at present, a symbol rather than a person.

Göttingen is today a thriving industrial and university town with the usual problems of housing and parking which beset cities the world over. A map of the city shows the plan of the old town wall, which, during the course of centuries, has been transformed into a wooded and pleasantly landscaped promenade. A slight detour from this promenade takes one to an old cemetery, where one can find the carefully tended grave of Gauss. A little farther along on the Wall, one passes the Gauss-Weber monument. Though the town is dominated by the University, there is no central, steepled, general administrative building. The various units of the University are scattered all about the town, just as one finds streets in all sections of the town named after scientists who have been connected with the University. The mathematician will read with pride such names as David Hilbert Strasse, Felix Klein, Riemann, and, of course, Gauss Strasse.

The Mathematical Institute is housed in a modern building on Bunsen Strasse. It was erected with the aid of a grant of $275,000 from the Rockefeller Foundation about twenty-five years ago. In the library of the Mathematical Institute one cannot fail to notice large empty spaces on the bookshelves. Upon inquiry one will learn that about eighty per cent of the books in the library were destroyed by an ironic accident at the end of World War II. For safekeeping during the war years the books had been stored deep in a mine near Göttingen. At the end of the war, when the books were being brought to the surface, an explosion of unknown origin and accompanied by fire destroyed the most precious of the library's possessions.

However, it is not on Bunsen Strasse where one will find Gauss memorabilia. Although Gauss is always called one of the greatest mathematicians of all time, his post at the University of Göttingen was that of Director of the Observatory. In early documents, the Observatory is described as being merely "near" Göttingen. The city has spread out so that it now includes the Observatory in one of its residential districts. At the Observatory one can see a strongbox containing some of Gauss's personal possessions, a display of items connected with the invention of the electric telegraph, and the most well-known of all the portraits of Gauss.

The distance from the Observatory to the Johanniskirche is about a mile and a half. The church is used as a place of worship today, and a few students live in one of its towers. The plaza on which the church stands serves as a parking lot and as a setting for the open-air market which comes to town several days during the week. Weber's physics laboratory was destroyed many years ago to make way for

the State and University Library on Prinzen Strasse, a block away from the Johanniskirche. Next door to the State and University Library is the Mercur Bank, a successor of the bank in which Reuter became a messenger boy in 1829. Mr. R. E. Benfey, a grandson of Siegfried Benfey, the uncle who had befriended young Josephat when he first came to Göttingen, directed the affairs of the bank until his retirement about twenty-five years ago.

After so many years, would the mathematics teacher of today on a visit to Göttingen be able to see some contemporary document, letter, or photograph—some concrete evidence of an association, however remote, between Reuter and Gauss? There remains a brief memo written by Gauss confirming a financial transaction with the bank. This note, which has been carefully preserved in Mr. Benfey's home, is the oldest record in the bank's files.

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On the practicality of the rule of false position

by Howard Eves, University of Maine, Orono, Maine

In the Rhind (or Ahmes) papyrus, which dates back to at least 1650 B.C., we find the following problem: "A quantity, its 2/3, its 1/2, and its 1/7, added together, become 33. What is the quantity?" Today, with our algebraic symbolism, we would probably solve this problem by letting x denote the sought quantity. Then we have

$$x+2x/3+x/2+x/7=33,$$

which, upon solving, yields $x=1386/97$.

But how would an Egyptian of 1650 B.C. solve this problem? The procedure for solving linear equations employed in those early days was a form of the method later to become known as the *rule of false position.* The method, in brief, is to assume any convenient value for the desired quantity, calculate the sum of the required parts, and then "blow up" the assumed value in the ratio that will convert the calculated sum into the given sum. In the problem of the Rhind papyrus, for

example, we might assume the desired quantity to be 42 (this is convenient because 42 is divisible by 3, 2, and 7). Then we find $42+(2/3)42+(1/2)42+(1/7)42 = 42+28+21+6 = 97$, instead of 33. Since 97 must be multiplied by 33/97 to give the required 33, the correct value of the quantity must be 42(33/97), or 1386/97. It is stimulating to contemplate that from a pure guess of the answer we are able to obtain the true answer.

It must be confessed that there is a certain element of beauty and simplicity in the primitive rule of false position, and it is no wonder that this rule, or some modification of it, was used even as late as the nineteenth century. Without a suitable algebraic symbolism, what else could one do? Actually, the method has both pedagogical and practical value, for not only do students find it interesting, but there are instances where the procedure has decided advantages. Let us consider the following examples, the first three of which are taken from the *Greek Anthology* (*ca.* 500 A.D.).

1. After staining the holy chaplet of fair-eyed Justice that I might see thee, all-subduing gold, grow so much, I have nothing; for I gave 40 talents under evil auspices to friends in vain, while O ye varied mischances of men, I see my enemies in possession of the half, the third, and the eighth of my fortune. (How many "talents" did the unfortunate man once possess?)

Suppose the man's original fortune had been 24 talents. Then $24-24/2-24/3-24/8 = 1$, instead of 40. Therefore the man once possessed (24)(40), or 960, talents.

2. Brickmaker, I am in a hurry to erect this house. Today is cloudless, and I do not require many more bricks, for I have all I want but three hundred. Thou alone in one day couldst make as many, but thy son left off working when he had finished two hundred, and thy son-in-law when he had made two hundred and fifty. Working all together, in how many days can you make them?

Suppose the required time is 1/5 of a day. Then, in this time, the three men can make $60+40+50$, or 150, bricks. Since 300 bricks are needed, the answer is 2/5 of a day.

3. I am a brazen lion; my spouts are my two eyes, my mouth, and the flat of my right foot. My right eye fills a jar in two days [1 day = 12 hours], my left eye in three, and my foot in four. My mouth is capable of filling it in six hours. Tell me how long all four together will take to fill it.

If the time were 12 days, then the right eye would fill 6 jars, the left eye 4 jars, the foot 3 jars, and the mouth 24 jars, whence, in 12 days, all spouts together would fill 37 jars. Therefore, it takes 12/37 of a day, or 144/37 hours, for all spouts together to fill one jar.

4. Find the length of BC in the quadrilateral pictured in Figure 1.

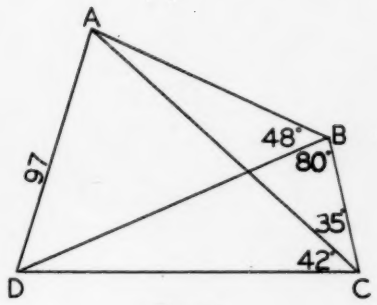

Figure 1

Apply the rule of false position by taking BC to be any convenient length, say 1 unit. Then, by the law of sines, we find $CD = 2.52$ and $AC = 2.69$. Next, by the law of cosines, we find $AD = 1.88$, instead of the given 97. It follows that $BC = 1(97/1.88) = 51.7$. This is a highly practicable procedure for solving this problem, and is actually the method a surveyor would probably use.

5. In the study of geometrical constructions there is a counterpart of the rule of false position, generally known as the

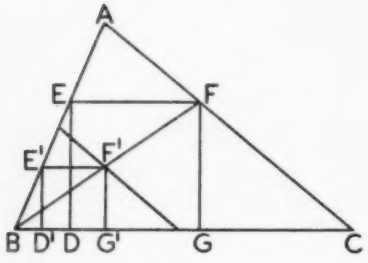

Figure 2

method of similitude. The method lies in constructing a figure similar to the one desired, and then, by the use of proportion, "blowing it up" to proper size. Suppose, for example, we wish to inscribe a square $DEFG$ in a given triangle ABC so that side DG of the square lies along the base BC of the triangle. (See Figure 2.)

Choose any point D' on base BC and draw the square $D'E'F'G'$, where E' is on BA and G' is on BC. Now, using B as a center of similitude, blow up the square $D'E'F'G'$ to the proper size by projecting F' into F on AC, etc.

Logs—Laws of Operation

In place of the three or four italicized operational laws of logarithms, my students prefer the package deal:

$$\log_a A \cdot B^n = \log_a A + n \cdot \log_a B$$

The derivation rests upon the assumption (of course) that two numbers exist for positive A, B such that $a^x = A$, $a^y = B$. Thus $a^{ny} = B^n$ and $AB^n = a^{x+ny}$ or $ny = \log_a B^n$ and $x + ny = \log_a AB^n$. Accordingly,

$$\log_a AB^n = x + ny = \log_a A + n \cdot \log_a B.$$

Familiar special laws occur for $A = 1$, $n = 1$, $n = -1$.—*Robert C. Yates, Williamsburg, Virginia.*

Some facts about top talent

Newsletter #108 (October 4, 1957) raised a question concerning the fate of high school students who failed to win an award from the National Merit Scholarship Corporation's first scholarship program in 1956.

Of the estimated senior class population of 1,196,500 of the country, the top five per cent were selected to take the first competitive examination. Of the 58,158 participants 556 were granted scholarships. An additional 4226 for whom funds were not available were awarded Certificates of Merit in recognition of their abilities.

The question we raised concerned the 57,602 who did not receive scholarships, and unfortunately their fate has not been yet determined. However, there is now valuable information available on the fate of a part of this group, the 4226 recipients of the Certificates of Merit.

The information was secured by means of a survey conducted ten months after the awards were made. Responses were obtained from 73% of the group of 4226. One of the striking facts revealed was that only 40 of the respondents (1%) reported that they were not then attending college. Of these, 19 expected to attend in the near future. Their failure to attend college was explained as follows: entering military service (7), illnesses and accidents (3), family and personal problems (2), accidental death (1), travel (1), engagement (1), and no information given (4).

Of the responding group 65% of the men received some scholarship help and another 27% received other forms of financial aid. No scholarship aid was given to 33% of the group.

Although we are not in any way suggesting that these statistical findings have direct extrapolatable national implications, it is remarkable to note that where a valuable mechanism of talent identification was provided, considerable evidence was accumulated showing the highest possible current rate of utilization of our most talented high school students.—Engineering and Scientific Manpower Newsletter, *January 13, 1958.*

Edited by Lucien B. Kinney, Stanford University, and Dan T. Dawson, Stanford University, Stanford, California

Mathematical contributions to the needs of youth

by Chester Scott, California State Polytechnic Institute, San Luis Obispo, California

In examining the organized efforts to bring the junior high school mathematics curriculum into focus with its proper functions, two movements become apparent: the administrative reorganization which created the junior high school, and the attempts to redirect the objectives of junior high mathematics.

What the junior high school is for

Around the turn of the century considerable dissatisfaction developed with the 8–4 plan of the elementary and high school program. Many forces were responsible for this feeling: compared to European practices there was too long a delay before our children began their secondary education; the mortality rate after the sixth grade was entirely too high; there was inadequate provision for the wide variation of capacity, interests, and needs of children in and out of school; the existing school program was not adequately distributing young people to occupational life and to opportunities for further training. At about age 12 the average student is rapidly approaching young adulthood. He is going through changes which make him unsuited for many of the experiences in elementary school.[1]

Gruhn and Douglass state that "educators at the elementary and secondary levels became immediately aware of this dissatisfaction and soon were participating actively in the investigations designed to improve the educational program below the college level."[2] Out of all the proposals the 6-3-3 plan was the one which caught fire and spread most rapidly.

What the mathematics curriculum is for

A variety of organized efforts have been made to ascertain the social needs of the curriculum and to fit the curriculum to these needs. The following efforts can be cited: Committee on Economy of Time (1890), Committee of Ten (1894), Committee on College Entrance Requirements (1899), Committee of Fifteen (1908), and the National Committee on Mathematical Requirements (1916). The Committee on Reorganization of Secondary Education published in 1918 the now famous "Seven Cardinal Principles." In 1936, the Educational Policies Commission outlined four areas in which objectives are to be set up. The Joint Commission to Study the Place of Mathematics in Secondary Education, created by the National Council of Teach-

[1] L. V. Koos, *The Junior High School* (New York: Ginn and Company, 1927), pp. 1 and 2.

[2] W. T. Gruhn and H. R. Douglass, *The Modern Junior High School* (New York: The Ronald Press Co., 1947), p. 11.

ers of Mathematics and the Mathematical Association of America, listed (1940) the well-known general objectives of secondary education.

The most recent (1947) in these lists of purposes of education is the Educational Policies Commission's "Ten Imperative Needs of Youth." These needs are summarized below:

1. The need to develop salable skills.
2. The need to develop good mental and physical health.
3. The need to understand rights and duties of democratic citizenship.
4. The need to understand significance of the family for individual and society.
5. The need to know how to be an intelligent consumer.
6. The need to understand methods of science and their application to man and nature.
7. The need to have opportunities to develop their capacities to appreciate beauty in literature, art, music, and nature.
8. The need for a more worthy use of leisure time.
9. The need to develop respect for other persons and also to develop ethical, aesthetic, and spiritual values of life.
10. The need to grow in their ability to think rationally, express thoughts clearly, and to read and listen with understanding.

A close look at all four sets of objectives must tell the observing student that each one is stating about the same thing in a slightly different way. (Note that the last set is stated in more operational terms.) Assuming this to be a valid observation, the "Ten Imperative Needs of Youth" deserve further scrutiny.

Contributions of Mathematics to the Needs of Youth

To see how mathematics may assume its share of the responsibility of satisfying these needs, each need may be considered separately to determine the extent to which mathematics contributes to its fulfillment.

1. *The need to develop salable skills.* It goes without saying that the student who is preparing for a vocation or profession should provide himself with the background work necessary for his chosen vocation or profession. If he wishes to become a skilled tradesman, technician, businessman, engineer, or just about any professional man, he must learn those mathematical skills necessary to his duties. Mathematics is a *must* for these students.

2. *The need to develop good mental and physical health.* While mathematics probably can make no major contribution to this need, it should be pointed out that in the organization and planning of activities for the mathematics classroom, the teacher can provide an atmosphere in which students can acquire the confidence and personal growth essential to meeting this need.

3. *The need to understand rights and duties of democratic citizenship.* Such fields as social science, home economics, and industrial arts are commonly considered chiefly responsible for the primary satisfaction of this need. Yet mathematics can play its part. The mathematical analysis of real social and civic problems removes them from the realm of controversy and social and political bias. The facts of the problem can be observed and acted upon with much less local prejudice entering the discussion. A real mathematical and social understanding of tax problems lends dignity and rationality to their consideration. Cooperative study of current problems by the class can contribute effectively to the satisfaction of this need.

4. *The need to understand significance of the family for individual and society.* Again the satisfaction of this need has been assigned primarily to such areas as social science, industrial arts, and home economics, with a good "assist" from the English and physical education departments. Yet

it should not be forgotten that the family is the basic economic and consumer unit of society. It encounters such problems as installment buying, insurance, and taxes, to name only a few. If it becomes insolvent, society suffers. A proper mastery of the mathematical operations of the family unit will make possible proper management of the family affairs and fulfillment of the family's important obligations to society.

5. *The need to know how to be an intelligent consumer.* If one is to derive the greatest economic satisfaction from his personal income, he must recognize that the economic and mathematical implications of properly running a home today are tremendous. The fact that most of the economic problems of the consumer today are quantitative is sufficient evidence alone to demonstrate that mathematics should be the principal contributor to the fulfillment of this need.

6. *The need to understand methods of science and their application to man and nature.* You have no doubt heard the statement that "mathematics is the language of science." When one considers that the scientist depends on mathematics for the development and verification of many scientific facts, he realizes more and more the meaning of the statement. If much of the background of science is mathematics, one must therefore conclude that to understand science he must know a substantial amount of mathematics.

7. *The need to have the opportunity to develop their capacities to appreciate beauty in literature, art, music, and nature.* Literature, art, and music properly taught contribute largely to the fulfillment of this need. Mathematics, too, offers a major contribution in the field of aesthetic appreciation. The elementary pupil demonstrates this appreciation clearly. Continued experiences with rhythm, proportion, balance, symmetry, harmonics, and progressions are readily available in our culture as well as in nature to carry these areas of appreciation into adult life. The fact that many people enjoy these aesthetics in our culture, and at the same time dislike mathematics, is a strong indication that we are missing some wonderful opportunities in the teaching of mathematics in our schools today.

8. *The need for a more worthy use of leisure time.* Our leisure-time activities are becoming more and more quantitative. One has only to look at the statistics on the sports page to verify this. The use of problem material from such sources is an obvious way to draw on the interests of the age group. The use of recreational materials in the classroom is also valuable in improving the learning situation. Mathematical party games and problems in the newspapers and magazines open up a new source of enjoyment for many people.

9. *The need to develop respect for other persons and also to develop ethical, aesthetic, and spiritual values of life.* Mathematics, along with all other departments in the school, must cooperate to satisfy these needs. Many mathematical units can be planned to provide committee and group activities that call for democratic teamwork. These will help to develop proper attitudes as well as abilities. It is these attitudes, which sometimes seem to be incidental to or byproducts of our teaching, that are so important to successful participation in a democratic society. From these attitudes we can properly develop our ethical, aesthetic, and spiritual values of life.

10. *The need to grow in their ability to think rationally, express thought clearly, and to read and listen with understanding.* Ability in the scientific problem-solving process is the one thing most likely to fulfill this need, and this is the fundamental aim of mathematics. The solution of any scientific problem can be divided into six steps:

a) The recognition that a problem exists and a desire to solve it
b) Formulating a purpose

c) Collecting the data
d) Organizing and classifying the data and stating the hypotheses
e) The testing of the hypotheses
f) Drawing conclusions.

It seems evident that this problem-solving process requires one to think rationally, to express his thoughts clearly, and to read and listen with understanding.

Conclusions

The foregoing discussion indicates that mathematics has a central responsibility in the junior high school curriculum. It is not a collection of detailed skills, suitable for drill methods alone. The mathematics curriculum calls for a thoughtful study of problems that effectively use arithmetic, aided by some algebra and geometry of form, size, and position. This may well include study of such topics as home owning, installment buying, insurance, taxes, mortgages, investments, and home budgeting, at the level of student experiences. One can be assured that any program that is really effective will make intelligent use of all of the three fields of mathematics. The narrow concept of "perfecting the skills" must be replaced with a broader and more ambitious set of aims.

One such set of aims was presented by the Commission on Post-War Plans when they suggested that the junior high school should "come to grips with its dual responsibility, (1) to provide sound mathematical training for our future leaders of science, mathematics, and other learned fields, and (2) to insure mathematical competence for the ordinary affairs of life to the extent that this can be done for all citizens as a part of a general education. . . ."[3]

The Junior High mathematics teacher has a direct responsibility to keep the present school programs adjusted to the purpose for which it is designed. Mathematics teachers, the challenge is yours!

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[3] Commission on Post-War Plans of the National Council of Teachers of Mathematics, "Second Report of the Commission on Post-War Plans," The Mathematics Teacher, XXXVIII (May, 1945), 195.

The following item is a selection from the world's first encyclopedia, a compendium of what men knew or believed, produced by a Spanish bishop, Isidore of Seville, in the seventh century.

Number is divided into even and odd. Even number is divided into the following: evenly even, evenly uneven, unevenly even, and unevenly uneven. Odd number is divided into the following: prime and uncompounded, compounded, and a third class that comes between which in a certain way is prime and uncompounded but in another way is secondary and compounded.—*Taken from* A History of Education *by Luella Cole.*

• NEW IDEAS FOR THE CLASSROOM

Edited by Donovan A. Johnson, University of Minnesota High School, Minneapolis, Minnesota

In-service institutes for mathematics teachers

by Paul Jorgensen, Carleton College, Northfield, Minnesota

The present proposals for revision of the mathematics curriculum of the secondary schools of America call for an extensive program of in-service education for secondary school mathematics teachers. It is the responsibility of our institutions of higher learning to provide such education, and to do so without delay. What follows is a description of just such a program.

During the 1957–1958 academic year, Carleton College conducted an In-Service Institute for Mathematics Teachers supported by a grant from the International Nickel Company. The purpose of the institute was to bring to a group of mathematics teachers some of the elements of modern mathematics, particularly as these apply to the teaching of mathematics in the junior and senior high school, and to provide an opportunity for high school and college faculties to become more aware of each other's interests and problems.

The participating teachers were chosen, as far as possible, on the basis of ability and leadership in the teaching of mathematics. Although it was recognized that need will eventually be the primary consideration in selection, it was felt that more could be accomplished of immediate and long-range benefit if some of the leaders in secondary mathematics education in the area were chosen for the first of such institutes. A representative group of 15 teachers was selected, including seven junior high and eight senior high teachers.

The institute convened for 26 meetings spread over the entire academic year. All sessions were held on Thursday evening and lasted from 6:00 to 9:30 in the evening. Each meeting started with dinner. Following dinner, the guest for the evening spoke briefly on a topic related to his special interest. These dinner hours made it possible for all concerned to become well acquainted and provided an excellent opportunity for interplay of ideas and opinions between college and secondary school personnel. The members of the staff participating in the institute are quite firmly convinced that the dinner meeting was a most important part of the program and contributed in a large measure to any success the institute may have had.

The group then adjourned to the classroom for a two and one-half hour session of informal lecture and discussion. The mathematics session used the materials developed by the University of Illinois Committee on School Mathematics for basic text materials, with supplementary readings in Felix Klein's *Elementary Mathematics from an Advanced Standpoint*[1] and H. A. Thurston's *The Number System*.[2] The class sessions were devoted to a study of the entire curriculum developed by the Illinois Committee and to related mathematical concepts. The discussions were

[1] Felix Klein, *Elementary Mathematics from an Advanced Standpoint*, translated by E. R. Hedrick and C. A. Noble, 2 vols. (New York: The Macmillan Co., 1932 and 1939). Reprinted by Dover publications.

[2] Hugh A. Thurston, *The Number System* (London: Blackie, 1956).

lively and often developed into consideration of fundamental mathematical concepts or varied methods of presentation. In this way all participants contributed a great deal to the deliberations of the groups.

On completion of the institute each participant received a certificate, and undergraduate credit was granted to those who requested it and passed a final examination on the material discussed.

The budget provided funds for the dinners. Each driver was paid travel expenses at the rate of seven cents per mile. Car pools were arranged in order to keep the travel expenses at a minimum, thus allowing for more participants. All texts and materials were provided for each participant, including a complete set of the Illinois materials. No tuition was charged. Honoraria were paid to the director and assistant director. The entire expense of a field trip to observe the Illinois program at Urbana for two days was also paid by the Institute.

Among the significant outcomes of the institute, the following are the most evident to this observer. The institute

1. provided excellent opportunity for free and open discussion of problems between secondary school people and college people;
2. provided the teachers with an opportunity to learn some of the new mathematical concepts being recommended for inclusion in the secondary mathematics curriculum;
3. prompted two of the schools represented to start experimental programs using the UICSM materials under the direction of UICSM;
4. created a great deal of interest and enthusiasm for curriculum study and revision on the part of the participating teachers;
5. created further interest at Carleton in the area of in-service education for high school teachers.

Comments by the participants concerning the effectiveness of the institute reveal further outcomes, as evidenced by the following statements written to the director:

"I feel that this program aids me in enriching my present program in high school mathematics."

"I hope that next year we will be able to revise our mathematics curriculum so that more of the modern mathematics concepts I have learned here can be taught."

"The study of the Illinois approach to algebra has been interesting and stimulating. I think it is well that we older teachers stop and reevaluate our work and make an honest effort to improve our technique."

"Courses of this type should definitely be extended to a larger number of colleges."

The final comment above prompts the following concluding remark. The responsibility rests with the colleges, the money is available, and the teachers are vitally concerned. Who's next?

Bridget, an algebra card game

by Donovan A. Johnson

Bridget is a card game that can develop in your algebra class a climate which is favorable to learning. By adding variety and competition, it will provide practice that builds accuracy and efficiency in evaluating algebraic expressions. A deck can easily be made up with 3″ by 5″ cards.

Bridget is an algebra card game for four players similar to bridge. The deck consists of 40 cards. Each of the first eight

cards has a monomial expression, such as $5x$, $-2x$, or x^2, written in black. The next eight cards have binomial expressions, such as $(x+5)$, $(7-x)$, or $(3x-2)$, written in green. Eight cards have fractions, such as

$$\frac{5}{x}, \quad \frac{x+2}{3}, \quad \text{or} \quad \frac{x}{5-x},$$

written in red; eight have mixed expressions, such as $\sqrt{x}$, $2x$, or πx, written in blue. The remaining eight cards are value cards. These should be different in color or shape from the 32 playing cards. Each value card gives a value of x, such as $x=2$, $x=-1$, or $x=0$.

Each player is dealt eight cards. The eight value cards are placed face down on the center of the table. The dealer starts the game by turning up the top value card. The player at the left of the dealer then plays any card of any color. As the play passes to the left, each person plays one card of the same color. The card with the highest value of the color led takes the trick, which consists of the four cards played. The value of each card is determined by evaluating the algebraic expression on each card, substituting for x the number from the value card. If a player cannot play the color led, he may play any other card in his hand. However, a card of different color cannot take a trick.

Play continues with the dealer turning up the next value card and the player who took the previous trick leading with a card of his choice. In case of ties, the trick is awarded to the next person winning a trick. When a person cannot evaluate his card or gives an incorrect value he should not win the trick. The score for each player, determined after all cards have been played, is the number of cards he has taken.

The game can be made more complex by using a system of bidding and declaration of trump-color similar to that used in bridge. Still another scoring system can be used which counts black cards one point, green cards two points, red cards three points, and yellow cards five points. The deck that is used can be made simpler or more complex, depending on the ability and maturity of the students. For example, to make a more complex deck use logarithms, trigonometric functions, fractional exponents, and progressions.

A complete set of cards for ninth-grade algebra can be made using these values for x: $1, 2, 3, 5, 10, -1, -2, \frac{1}{2}$. Expressions for substitution may be the following:

1. Monomials (black)—$2x$, x^2, $-3x$, $+5x$, x^3, $-2x^2$, $-x$, $-4(-2x)$;
2. Binomials (green)—$x+4$, $2x+5$, $x-5$, $7-2x$, x^2+5, $10-x^2$, x^2+3x, $(x-3)^2$;
3. Fractions (red)—$\frac{x}{2}$, $\frac{x^2}{3}$, $\frac{5}{x}$, $\frac{2x}{3}$, $\frac{x+3}{5}$, $\frac{5}{2x}$, $\frac{3}{x+3}$, $\frac{3-x}{5}$;
4. Mixed expressions (blue)—πx, $\pi - x$, $\sqrt{x+3}$ $(-x)^2$, $-x^2$, x^2-3x, $\frac{1}{x}$, $10-4x$.

> Although indoctrination, along with training, is the overriding goal of the Soviet educational system, there is ample evidence that its success in formal training in literary and manual skills, technology, engineering, science, and, notably, in mathematics, has not been matched by anywhere near equal success in Communist indoctrination.—*Alexander G. Korol*, Soviet Education for Science and Technology, *p.* 411.

A column of unofficial comment

The school mathematics study group

by E. G. Begle, Yale University, New Haven, Connecticut

The world of today demands more mathematical knowledge on the part of more people than the world of yesterday, and the world of tomorrow will demand even more. It is therefore important that mathematics be taught in a vital and imaginative way which will make students aware that it is a living, growing subject which plays an increasingly important part in the contemporary world.

An awareness of present-day shortcomings, both in the teaching of mathematics and in the content of the courses taught, led several professional organizations to concern themselves with providing possible solutions to these serious problems.

The American Mathematical Society, the Mathematical Association of America, and the National Council of Teachers of Mathematics were highly instrumental in prevailing upon the National Science Foundation to provide the necessary funds to set up the School Mathematics Study Group, under the Executive Directorship of Professor E. G. Begle of Yale University.

The main purpose of the School Mathematics Study Group is to develop a curriculum and teaching materials based on the best available knowledge of mathematics, pedagogy, and the needs of our society. That this is a truly sizable task is understandable. The SMSG agrees with other groups that are concerning themselves with certain aspects of the same problem that proper mathematical instruction in our elementary and secondary schools is of the utmost importance in the scientific training of our young people. It further agrees that the present mathematics curriculum is out of phase with the actual needs of our students as well as with the developments within the field of mathematics itself.

The problem of providing a curriculum together with instructional material for all students, with their varied interests and abilities, is a tremendous task. Five points must be borne in mind in planning such a curriculum:

1. No one can predict exactly which mathematical skills will be important and useful in the future.
2. No one can predict exactly what career any particular student will choose when he leaves school.
3. Teaching which emphasizes understanding, insight, and imagination, without neglecting basic skills, is the best for all students, whatever their ability, and makes the best preparation for any vocation that uses mathematics.
4. An understanding of the role of mathematics in our society is essential for intelligent citizenship.
5. Any normal individual can appreciate some, at least, of the beauty and power

of mathematics, and this appreciation is an important part of a civilized person's cultural background.

The SMSG will present the combined efforts of professional mathematicians, teachers of mathematics, and teachers of teachers of mathematics. The resulting recommendations and materials should represent a healthy fusing of correct mathematics and sound pedagogy.

No single curriculum and no single text should ever be thought of as final and definitive. The SMSG in no way proposes this situation. It is, however, aware of the fact that curriculum revision not supplemented by sample material for classroom use will wither and die on the vine. Consequently, one of the major tasks of the group will be the preparation of sample texts written to the proposed curriculum. This will be done for grades 7–12.

The first formal session of SMSG was held at Yale University from June 23 to July 18, 1958. The facilities of Trumbull College, on the Yale Campus, were made available to some forty persons representing universities, colleges, secondary schools, teacher-training institutions, and industry. From a geographical point of view, representation was widespread, with people from both coasts as well as the other major areas attending the session either full- or part-time.

Before arriving, each participant was sent literature from other groups which have been and still are active in the field of curriculum revision. The first day was spent not only in organizing for the rest of the month but also in being thoroughly briefed by members of the groups mentioned above.

Five subgroups were formed. Four of these devoted their efforts to grades 9 through 12, while the fifth concerned itself with grades 7 and 8. Each group was comprised of representatives from the colleges as well as the schools. Formal coordinating meetings were held during the session in order to insure the fact that all groups were aware of each other's problems and progress. The coffee breaks, meals, and informal evening discussion groups afforded excellent opportunities for the mutual give-and-take of ideas so essential in a task such as confronts SMSG.

The SMSG feels that it is particularly important that greater substance and interest be given to the mathematics of grades 7 and 8. The subgroup concerned with these grades prepared a number of units, chosen for their mathematical value, their interest to students, and their importance in general education. These, as well as other similar units prepared by the University of Maryland Junior High School Project, with which the SMSG is co-operating, will be tried out in a variety of classes during the coming year. From those which prove both valuable and teachable, a new curriculum will be prepared at a conference to be held in the summer of 1959.

The subgroups concerned with grades 9 through 12 had more to start with. To correlate their efforts to some extent, each of these subgroups started with the recommendations of the Commission on Mathematics. Each drew up a detailed outline for a text for its grade, and in some cases complete chapters were written. Additional chapters will be written during the coming academic year, and it is hoped that complete manuscripts for all four grades can be completed during a second writing session in the summer of 1959.

That so much could be accomplished in one month is a direct tribute to the ability of able mathematicians and experienced classroom teachers to understand each other's problems and to communicate with each other so lucidly about them. The flavor of the entire session was one of mutual understanding, co-operation, admiration, and an eagerness to produce fine mathematics for future generations of students.

It would be safe to say that the student population aimed at in this preliminary work was composed of the better students

in the college-capable group. It seemed proper to the SMSG to consider this group first, since it is from this group that our future scientists and engineers, as well as our leaders in all the professions, will come for the most part. Nevertheless, the SMSG has no intention of neglecting the slower students. Quite often, at present, such students are shunted off to a course which deprives them forever of a chance to take further courses in mathematics. We feel that this is deplorable, and that all students should have an opportunity to take as much good mathematics as they are capable of. In theory, the course designed for the faster students should, if made perhaps more intuitive and taken at a slower pace, be attractive and valuable for slower students. Whether practice will accord with theory is a question which the SMSG will study very carefully. Such a study must await the completion of our sample texts, but it should be possible to make a start on them this next year.

In all of its deliberations the SMSG gave careful consideration to the work and proposals of such groups as the Commission on Mathematics, the UICSM, the New York State Regents, and other interested groups. It does not feel that it is in any way bound to accept any or all of the findings of any of these groups, but it does recognize that tremendous spade work has been done by these groups in the field of curriculum revision. Not to profit from these findings would be a serious error.

The role of films, film strips, and TV as useful teaching aids will be considered. Only after a great deal of careful study and experimentation with these media will any recommendations be made.

It is planned to stimulate outstanding mathematicians to write a series of monographs on such mathematical topics as: Non-Euclidean Geometry, Groups, Topology, Computing Machines, Logic, Number Systems, Cryptography, Statistics, etc.; these all to be directed toward the better students as well as the classroom teacher.

No group concerned with curriculum modification can ever evade the teacher-training issue. Here again the SMSG proposes to make its suggestions only after careful deliberation. But it does plan to face up to the issue.

It is the considered opinion of the group that if the busy and dedicated classroom teacher of mathematics is given a mathematically and pedagogically sound text, she will willingly, even eagerly, teach good mathematics to her students. With no such text, even the devoted teacher is all too frequently lost.

And so a combination of experienced classroom teachers and able mathematicians have dedicated themselves to producing sound texts, directed toward a sound mathematics curriculum, as well as the many corollaries of this charge. The product of this education can only prove to be a tremendous asset to the future of America.

The cultivation of the mathematical imagination depends chiefly on the child being put into the right attitude towards mathematical conceptions in his earliest years; and, after that, on the right use being made of certain nodes or critical points, which occur here and there in each branch of mathematics and which should be dealt with in a quite different manner from the rest of the course.—*Mary Everst Boole in* Preparation of the Child for Science.

• TESTING TIME

Edited by Robert S. Fouch, Florida State University, and Robert Kalin, Florida State University, Tallahassee, Florida

Grouping, standardized tests, the Education Act: a miscellany of information

by Robert Kalin

Many things went "up" along with the first Russian Sputnik in the fall of 1957. Dozens of newspaper articles soon reflected a sudden rise of interest in education among the American public, with mathematics and science courses receiving a major share of attention. In the first full academic year since Sputnik I, many teachers are already reporting enrollment increases in nonrequired mathematics courses.

Teacher reports also indicate a renewed interest in ability or achievement grouping in mathematics; seemingly many school systems have taken steps to separate out for special instruction their mathematically more talented students. As this apparent trend develops, identification techniques will become most important. These will undoubtedly include at least some combination of grades, teacher judgments, and standardized tests.

In considering possible standardized tests to use in this connection, mathematics teachers and their colleagues in guidance will without doubt consult the "Mental Measurement Yearbooks" edited by Professor O. K. Buros. Latest in this series is *The Fourth Mental Measurement Yearbook*, published in 1953 by the Gryphon Press of Highland Park, New Jersey.

The essential contents of each *Yearbook* are these:

1. reviews of tests of many types (achievement, ability, personality, etc.) and in many subject areas (English, foreign language, mathematics, science, etc.)
2. reviews of text and reference books in the testing field
3. directories of periodicals and test publishers
4. indexes of titles, names, and tests (classified).

Here a school system would find guidance in deciding which type(s) of tests might be most suitable. Or more particularly, if a decision had been made to achieve a grouping via some kind of algebra achievement test, for instance, the test reviews in the appropriate section of a Buros' *Yearbook* would list most of the available tests, then give the investigator some initial guidance as to which test(s) might serve best.

Also extremely handy is the "Publishers Directory" in each *Yearbook*. For, after some initial screening, teachers will perhaps choose to send for sample examination copies so that a final intelligent decision can be made on the basis of a close analysis of test content and associated materials.

Since the publishers directories in Buros' *Yearbooks* are quite lengthy, perhaps containing more information than is immediately pertinent for the mathematics area, a list is provided here of the names and addresses of some test publishers who publish or sell tests relevant to mathematics education, and who have catalogs or other potentially helpful materials:

California Test Bureau, 5916 Hollywood Boulevard, Los Angeles 28, California
Cooperative Test Division, Educational Testing Service, 20 Nassau Street, Princeton, New Jersey
Educational Test Bureau, Educational Publishers, Inc., 720 Washington Avenue, S.E., Minneapolis, Minnesota
C. A. Gregory Company, 345 Calhoun Street, Cincinnati 19, Ohio
Houghton Mifflin Company, 2 Park Street, Boston 7, Massachusetts
Psychological Corporation, 522 Fifth Avenue, New York 36, New York
Public School Publishing Company, 204 West Mulberry Street, Bloomington, Illinois
Science Research Associates, 57 West Grand Avenue, Chicago 10, Illinois
World Book Company, Yonkers-on-Hudson, New York

Teachers who wish to refresh their knowledge of testing theory—construction, scoring, analysis, etc.—will find the "Books and Reviews" section of Buros' *Yearbooks* a helpful guide. Again, however, the somewhat lengthy searching required for mathematics teachers to discover relevant items makes a few suggestions in order. For those who wish to consider such a topic as measuring understanding in mathematics, two chapters in Part I of the *45th Yearbook* of the National Society for the Study of Education should prove of prime interest. This volume (*The Measurement of Understanding*, published by the Society in 1946, and distributed by the University of Chicago Press) contains a chapter authored by Messrs. Hartung and Fawcett in which are indicated some ways of writing high school mathematics test questions that should measure more than sheer computational skill. The preceding chapter, written by Miss Sauble and Messrs. Sueltz and Boynton, deals with elementary school mathematics in an analogous fashion.

Of wider scope is the text *Educational Measurement*, edited by E. F. Lingquist and published by the American Council on Education in 1951. Here one may study recommendations by outstanding test theorists on such matters as planning, writing, pretesting, administering, and scoring tests.

Although the intricacies of the National Defense Education Act of 1958 have yet to be untangled as this column goes to press, it seems clear that monies will be made available to support high school testing programs in the various subject areas, including mathematics, since the purpose of that section of the bill dealing with guidance matters is to guarantee that students be properly counseled into courses of study appropriate to their talents. The money is considerable: Congress has authorized $15 million for each of the next four fiscal years, to be allocated to the various state departments of education in proportion to each state's fraction of the national school-age population. In addition, each state must match federal grants after the first year.

It would seem that a portion of this money may be used to support intelligently planned local testing programs in mathematics. High school mathematics departments that have in the past failed, for financial reasons, to adopt such programs would seem well advised to consult their state department of education for further information.

Junior's Destiny

From indications that we see,
We feel safe in foretelling
That he'll go down in history . . .
And also math and spelling.
—*Stephen Schlitzer*

• TIPS FOR BEGINNERS

Edited by Joseph N. Payne, University of Michigan, Ann Arbor, Michigan and William C. Lowry, University of Virginia, Charlottesville, Virginia

The metric system in junior high school

by Isabelle P. Rucker, State Department of Education, Richmond, Virginia

When children enter the eighth grade they are generally eager, interested, and anxious to embark on a new phase of their education. Their enthusiasm for mathematics wanes when the content is only a review of the arithmetic they had in the earlier grades. I would like to pass on some suggestions for a unit in eighth-year mathematics that has proved to be a very successful and motivating experience for our students.[1]

Early in the year I introduced a unit on measurement—but measurement using the units of the metric system. The metric system is, of course, valuable content for the pupils. And it has the additional advantage of being "something new." Although I have used the unit in grade eight, it could be adapted easily to grade-seven or grade-nine general mathematics.

Before the unit begins, I assemble materials pertaining to the topic and make sure that the décor of my classroom provides the appropriate setting for the study. My materials include the following:

1. Metric Wall Chart, available from the U. S. Government Printing Office, Washington 25, D.C., for about 50 cents.
2. Pictorial History of Measurement charts, available free from The Educational Division, Ford Motor Company, Detroit, Michigan.
3. A wall chart of word derivation pertinent to the metric system. This chart I made myself the first time, but now I use one that a former student made. The chart has the words:

   MILLI—means 1/1000
   CENTI—means 1/100
   DECI—means 1/10
   DEKA—means 1000
   KILO—means 10
   (one mil = 1/1000 of $1.00)
   (one cent = 1/100 of $1.00)
   (one dime = 1/10 of $1.00)
   (one decade = 10 years)
   (one kilowatt = 1000 watts).
4. A bulletin board display of newspaper articles (Olympic games, etc.), pictures and sections from old physics books, strips of old movie film, both 35 mm. and 16 mm.
5. A road map of a European country which shows mileages in the metric units of measure.
6. A meter stick with well defined and easily distinguishable markings.

Each child is asked to purchase two things: (1) a ten-cent plastic ruler with English units (inches) on one edge and metric units (centimeters and millimeters) on the other edge; and (2) a packet of metric graph paper. Since our school is in a rural area, I find it easier to purchase

[1] This article was written while Mrs. Rucker was teaching at Louisa County High School, Mineral, Virginia.

these items in quantity and sell them to the students. It assures me that everyone has been given a chance to acquire the necessary equipment. If a student comes to class without his instruments, he may rent some for that class period.

Some of the activities of the unit are explained in the remaining paragraphs. It should be noted here that throughout the unit I make frequent reference to the parallel between our number system and the metric system of measurement. Our number system based on 10 helps pupils understand the metric system of measurement, also based on 10. On the other hand, the study of the relationship between units of the metric system strengthens and broadens the pupil's concept of place value in the number system. For example, 1 kilometer is 1000 meters; in the numeral 3003, the 3 at the left represents a value 1000 times the value represented by the 3 at the right. Similar analogies between the number system and the metric system of measurement are an integral and vital part of the unit.

The first day we talk about how the metric system was established in France, and we find out that the meter is the standard measure in the metric system. I ask if anyone in the class has been to the National Bureau of Standards in Washington, D. C. I point out that the meter bar at the Bureau is made of platinum and iridium and is a precise duplicate of the original meter bar in Paris. The material of which the bar is made is interesting to only a few students at this stage. But some of them may do some reading and find out that these metals are used because they are less affected by weather changes than other metals.

We look at a long mark which was drawn on the blackboard before class and we call its length a meter. We see that a meter is a bit longer than a yard, by about three inches. We examine the demonstration meter stick and see that it is divided into ten main sections, or ten smaller units. From the prefix chart we see that this smaller unit is called a decimeter, since it is one-tenth of a meter. We notice that the next smaller unit on the meter stick is one one-hundredth of the meter and that it is called a centimeter. Finally, we take the smallest unit on the meter stick, decide how many of these it takes to make up the meter, and call it the millimeter. At this point I give each student a strip of either 35 mm. or 16 mm. film, obtained from the local movie house, and let him measure its width with his ruler. Most students have heard of the terms "35 mm." and "16 mm.," but few realize that these are measurements in the metric system. Graph paper is very useful to help visualize the lengths of these metric units.

With this broad idea of the meter as a unit of measurement, we start again with the smallest unit on the meter stick, the millimeter, find it on our plastic ruler, and draw its length on paper. We use the ruler, the graph paper, and then, by estimating, draw its length freehand. This same procedure is followed with each of the units smaller than the meter until the students can quickly draw, freehand, a line segment of any specified number of units in length with reasonable accuracy. Then we start to measure things, using the metric units of linear measure only. We measure everything in sight, from the length of the thumbnail to dimensions of the desk top, and even measure the heights of one another. Eighth graders like this!

If the students have had any background in finding areas, we use the formulas with which they are familiar and find areas, using the square units of the metric system. Here again the graph paper is a fine learning aid. If, however, the background of the pupils is not so extensive, we use the units of the metric system in well chosen, previously prepared examples which involve the various operations of arithmetic. In so doing, the students are learning about "something new" and very subtly they are polishing skills in which they have become rusty during the summer months. Review, *per se*, is often bor-

ing, but this doesn't have the stigma of "review."

Our study of the metric system enables the teacher to determine the mathematical status of each student, and it can aid in identifying the more able ones. If the class is a very astute one I do not stop with the linear measurements, but introduce units of weight, capacity, and volume.

The length of time that the unit is studied depends upon the class. I have found it to be interesting and stimulating for both teacher and students for at least two weeks. It does require, as does anything worthwhile, a good amount of thought and preparation in advance; and above all it requires that the teacher be quite familiar with the mathematics, the history, and the uses of the metric system.

What's new?

BOOKS

MISCELLANEOUS

Studies in the Mathematical Theory of Inventory and Production, Kenneth J. Arrow, Samuel Karlin, and Herbert Scarf. Stanford, California: Stanford University Press, 1958. Cloth, xii+340 pp., $8.75.

Table for the Solution of Cubic Equations, Herbert E. Salzer, Charles H. Richards, Isabelle Arsham. New York: McGraw-Hill, Inc., 1958. Cloth, xv+161 pp., $7.50.

Teaching Science to the Ordinary Pupil, K. Laybourn and C. H. Bailey. New York: Philosophical Library, 1957. Cloth, 416 pp., $10.00.

Working with Student Teachers, Florence B. Stratemeyer and Margaret Lindsey. New York: Bureau of Publications, Teachers College, Columbia University, 1958. Cloth, vi+502 pp., $4.75.

Understanding and Teaching Arithmetic, E. T. McSwain, Ralph J. Cooke. New York: Henry Holt and Company, 1958. Cloth, xi+420 pp., $5.50.

BOOKLETS AND MATERIALS

Analysis of Research in the Teaching of Mathematics, 1955 and 1956 (FS 5.3:958/4), Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. 73-page booklet written and compiled by Kenneth E. Brown, 25¢.

The Balance Sheet on Education, The Detroit Science Education Story, The Indianapolis Science Education Story, The Oklahoma Science Education Story, Thomas Alva Edison Foundation, Inc., 8 West 40th Street, New York 18, New York. Four booklets, each giving one or more of the addresses made at the Edison Foundation's Eighth Institute; single copies free to teachers (limited supply).

Concepts of Equation and Inequality: Sample classroom unit for high school algebra students, Commission on Mathematics, College Entrance Examination Board, 425 West 117th Street, New York 27, New York. 11-page booklet, 15¢ each.

Directed Practice in Algebra, Educational Service, Inc., Benton Harbor, Michigan. Collection of test sheets and practice exercise sheets prepared by Sterling C. Crim; teacher's manual contains all sheets and instructions, $3.00; 109 separate sheets are provided for students, ½¢ per sheet.

Education in Mathematics for the Slow Learner, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W., Washington 6, D. C. 36-page booklet written by Mary Potter and Virgil Mallory, 75¢.

Getting Things Done Through Local Action, The President's Committee on Scientists and Engineers, Washington 25, D. C. Kit of materials designed for use not by an individual but by a community to help it organize its resources to resolve its education problems; includes reference materials on improvement of science curricula, and many other materials; free for a community.

Guideposts for the Education of the Gifted, Bureau of Publications, Teachers College, Columbia University, 525 West 120th Street, New York 27, New York. Set of folders with different folders for administrators, teachers, students, and parents; written by Ruth Strang; 60¢ per set; copies of any of the four folders, $1.00 per 10 folders.

How to Use Your Library in Mathematics, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W., Washington 6, D. C. 6-page illustrated booklet written by Allene Archer, 40¢.

Investment Resource Materials, Peter Yacyk, Ridley Township High School, Folsom, Pennsylvania. 52-page booklet, $1.00 each.

Mathematics Charts, L. E. Christman, Yorkville, Illinois. Set of 12 charts dealing with the history of mathematics, $1.50 per set.

• NOTES FROM THE WASHINGTON OFFICE

by M. H. Ahrendt, Executive Secretary, NCTM, Washington, D. C.

National Science Foundation announces thirty-two Academic-Year Institutes for high school teachers of science and mathematics

The National Science Foundation announced today the award of grants totaling over $8,600,000 to 32 colleges and universities in support of Academic-Year Institutes designed to help high school science and mathematics teachers improve their subject-matter knowledge.

An estimated 1,500 high school science and mathematics teachers will be enrolled in the Institutes in the 1959–60 academic year. Each teacher will pursue a program of study in the sciences and mathematics planned especially for him and conducted by leaders noted not only for competence in their fields but also for skill in presentation. The grants will provide stipends of $3,000 each to approximately 50 teachers in each Institute. Allowances for dependents and travel will also be provided. Certain Institutes will provide an additional summer training program to make it more readily possible for teachers to fulfill the requirements for a graduate degree. Supplementary allowances will be provided for teachers participating in this extended program.

The Academic-Year Institutes Program is being expanded by the Foundation because of its success in helping science and mathematics teachers improve the quality of their own teaching through enabling them to keep abreast of the latest scientific developments. The influence of these teachers on others with whom they come in contact is such that it raises the general level of science instruction in the secondary schools and also provides motivation for students to undertake science careers.

Other functions of these Institutes are to encourage colleges and universities to give greater emphasis to subject-matter instruction for teachers already in service and to provide similar subject-matter emphasis for their programs for prospective teachers of science and mathematics.

A list is appended showing the institutions receiving grants and the directors of each Institute. *Applications by high school teachers for acceptance in the Institutes should be sent to the directors.* All selections will be made by the host institution, NOT by the National Science Foundation.

INSTITUTIONS RECEIVING GRANTS FOR ACADEMIC-YEAR INSTITUTES, 1959–1960

Mathematics program only

Boston College (Rev. S. Bezuszka, S. J., Department of Mathematics, Chestnut Hill 67, Massachusetts)

University of Illinois (Prof. J. Landin, Department of Mathematics, Urbana, Illinois)

University of Kansas (Prof. G. B. Price, Department of Mathematics, Lawrence, Kansas)

Louisiana State University (Prof. H. T. Karnes, Department of Mathematics, University Station, Baton Rouge 3, Louisiana)

University of Minnesota (Prof. C. Hatfield, Department of Mathematics, S. L. A., 119 Folwell Hall, Minneapolis, Minnesota)

University of Notre Dame (Prof. A. E. Ross, Department of Mathematics, Notre Dame, Indiana)

San Diego State College (Prof. J. E. Eagle, Mathematics Department, San Diego 15, California)

General programs in the sciences and mathematics

Arizona State College (Prof. A. T. Wager, Department of Physics, Tempe, Arizona)

Atlanta University (Prof. K. A. Huggins, Department of Chemistry, Atlanta 14, Georgia)

Brown University (Prof. E. R. Smith, Department of Education, 71 Brown Street, Providence 12, Rhode Island)

University of Colorado (Prof. W. E. Briggs, Department of Mathematics, Hellems Annex 318, Boulder, Colorado)

University of Georgia (Prof. J. J. Westfall, Department of Botany, Athens, Georgia)

Harvard University (Prof. E. C. Kemble, Department of Physics, Lyman Laboratory, Cambridge 38, Massachusetts)

University of Hawaii (Prof. J. J. Naughton, Department of Chemistry, Honolulu 14, Hawaii)

Iowa State Teachers College (Prof. R. A. Rogers, Department of Science [Physics], Cedar Falls, Iowa)

University of Michigan (Prof. F. D. Miller, Department of Astronomy, 1018 Angell Hall, Ann Arbor, Michigan)

University of New Mexico (Prof. W. Ivins, Department of Secondary Education, College of Education, Albuquerque, New Mexico)

University of North Carolina (Prof. E. C. Markham, Department of Chemistry, 210 Venable Hall, Chapel Hill, North Carolina)

University of North Dakota (Prof. J. D. Henderson, Department of Physics, Grand Forks, North Dakota)

Ohio State University (Prof. J. S. Richardson, Department of Education, 250 Arps Hall, Columbus 10, Ohio)

Oklahoma State University (Prof. J. H. Zant, Department of Mathematics, Stillwater, Oklahoma)

Oregon State College (Prof. S. E. Williamson, Department of Science Education, Corvallis, Oregon)

Pennsylvania State University (Prof. W. H. Powers, Department of Chemistry, 102 Sparks Building, University Park, Pennsylvania)

University of Pennsylvania (Dean W. E. Arnold, School of Education, Philadelphia 4, Pennsylvania)

State University of South Dakota (Prof. C. M. Vaughn, Department of Zoology, Medicine and Science Building, Vermillion, South Dakota)

Syracuse University (Prof. A. T. Collette, Department of Bacteriology and Botany, 400 Lyman Hall, Syracuse 10, New York)

University of Texas (Prof. R. C. Anderson, Department of Chemistry, Austin, Texas)

Tuskegee Institute (Prof. W. E. Belton, Department of Chemistry, Tuskegee Institute, Alabama)

University of Utah (Prof. T. J. Parmley, Room 215, Physical Science Building, Salt Lake City 12, Utah)

University of Virginia (Prof. J. W. Cole, Jr., Department of Chemistry, Charlottesville, Virginia)

Washington University (Prof. E. U. Condon, Physics Department, St. Louis 5, Missouri)

University of Wisconsin (Prof. D. H. Bucklin, Department of Zoology, Madison, Wisconsin)

Federal support for mathematics teaching

by Kenneth E. Brown, Specialist for Mathematics, Office of Education, U. S. Department of Health, Education and Welfare, Washington, D. C.

One of the most extensive legislative acts for the support of education in the history of our nation was passed during the last days of the 85th Congress.

The National Defense Education Act, 1958, authorizes an education program of 877 million dollars. Most of the cost will be shared by the Federal Government and the States.

Although no direct scholarships are provided, more than 300 million dollars is authorized for loans to undergraduate students and for postgraduate fellowships.

Of special interest to mathematics teachers is 300 million dollars in Federal aid to States for facilities and equipment to improve instruction in science, mathematics, and modern foreign languages. Congress authorized 70 million dollars a year for the next four years for making

payments to State educational agencies on a fifty-fifty matching basis for "(A) the acquisition of laboratory and other special equipment, including audio-visual materials and equipment and printed materials (other than textbooks), suitable for use in providing education in science, mathematics, or modern foreign languages in public elementary or secondary schools, or both, and (B) minor remodeling of laboratory or other space used for such materials or equipment."

An additional five million dollars a year (the funds are to be matched the last three years but not the first year) for four years was authorized for "(A) expansion or improvement of supervisory or related services in public elementary and secondary schools in the fields of science, mathematics, and modern foreign languages, and (B) administration of the State plan."

The purpose of the act is twofold: (1) to provide financial aid to needy students with ability and (2) to "correct as rapidly as possible the existing imbalances in our educational programs which have led to an insufficient proportion of our population educated in science, mathematics, and modern foreign languages and trained in technology."

To receive financial assistance for strengthening science, mathematics, and modern foreign language instruction, local schools will work with their State Education Agency to develop state plans for the U. S. Commissioner of Education to approve.

Your professional dates

The information below gives the name, date, and place of meeting with the name and address of the person to whom you may write for further information. For information about other meetings, see the previous issues of THE MATHEMATICS TEACHER. Announcements for this column should be sent at least ten weeks early to the Executive Secretary, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W., Washington 6, D. C.

NCTM convention dates

EIGHTEENTH CHRISTMAS MEETING

December 29–30, 1958
Sheraton-McAlpin Hotel, New York, New York
Elizabeth Sibley, 18 Stuyvesant Oval, New York 9, New York

THIRTY-SEVENTH ANNUAL MEETING

April 1–4, 1959
Baker Hotel, Dallas, Texas
Arthur W. Harris, 4701 Cole Avenue, Dallas 5, Texas

JOINT MEETING WITH NEA

June 29, 1959
St. Louis, Missouri
M. H. Ahrendt, 1201 Sixteenth Street, N. W., Washington 6, D. C.

NINETEENTH SUMMER MEETING

August 17–19, 1959
University of Michigan, Ann Arbor, Michigan
Phillip S. Jones, Mathematics Department, University of Michigan, Ann Arbor, Michigan

Other professional dates

Forty-second Annual Meeting, Mathematical Association of America

January 22–23, 1959
University of Pennsylvania, Philadelphia, Pennsylvania
Harry M. Gehman, University of Buffalo, Buffalo 14, New York

Mathematics Section of the New York Society for the Experimental Study of Education

February 14; March 14; April 17, 1959
Teachers College, Columbia University, New York, New York
John A. Schumaker, Montclair State College, Montclair, New Jersey

For information about the Academic-Year Institutes sponsored by the National Science Foundation for Teachers of Mathematics, see pages 624–625.

HOW TO—

NO. 1

How to Use Your Bulletin Board

Donovan A. Johnson
and Clarence E. Olander

Make your bulletin board a genuine teaching aid. Discusses purposes, appropriate topics, supplies needed, techniques, and "tricks of the trade."

12 pages. *50¢ each.*

NO. 2

How to Develop a Teaching Guide in Mathematics

Mildred Keiffer and Anna Marie Evans

Discusses steps in the development of a guide, principles involved, content, and use. Contains an annotated bibliography of reference materials.

10 pages. *40¢ each.*

NO. 3

How to Use Field Trips in Mathematics

Donovan A. Johnson, Dirk Ten Brinke,
and Lauren G. Woodby

Discusses types of field trips, purposes, where to go, how to plan, follow-up activities, and the like. Gives examples of actual field trips.

8 pages. *35¢ each.*

NO. 4

How to Use Films and Filmstrips in Mathematics Classes

Robert Vollmar and Philip Peak

Discusses the selection and use of films and filmstrips, including attitudes, objectives, preview and selection. procedures for effective use, and evaluation.

14 pages. *50¢ each.*

NO. 5

How to Use Your Library in Mathematics

Allene Archer

Discusses purposes, outcomes, reference materials, topics, and projects. Information on historical reports, things to make, mathematicians, quotations.

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III. Operating with Sets
IV. Deductive Methods in Mathematics
V. Algebra
VI. Geometric Vector Analysis and the Concept of Vector Space
VII. Limits
VIII. Functions
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XIII. Implications for the Mathematics Curriculum

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The Consulting Editors who will direct this new Series are dedicated teachers who have long been intimately associated with these problems. They are:

RICHARD S. PIETERS, *Chairman, Department of Mathematics, Phillips Academy, Andover, Massachusetts*

PAUL C. ROSENBLOOM, *Director, Minnesota National Laboratory for Improvement of Secondary School Mathematics and Professor of Mathematics at the University of Minnesota*

GEORGE B. THOMAS, JR., *Associate Professor of Mathematics, Massachusetts Institute of Technology*

JOHN WAGNER, *Consultant, Texas Science Teaching Improvement Program*

A number of texts are at present under development for the Science Education Series. Many of them are being tested in actual class use prior to formal publication. Announcements of these books will be made as they become ready for publication.

In the meantime, the publisher invites all who are interested in learning more about the Science Education Series to send for a detailed brochure.

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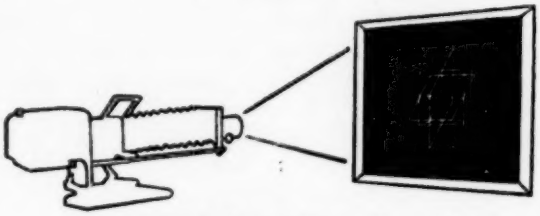
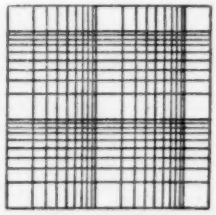
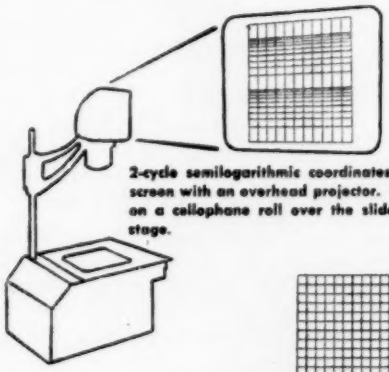


The 3-dimensional rectangular coordinates projected onto a chalkboard from a slide projector. Curves may be plotted with chalk at the board.



2-cycle log-log coordinates as they appear on the screen from an overhead projector.



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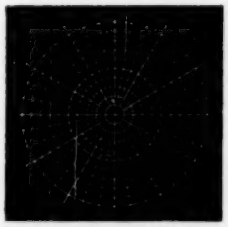

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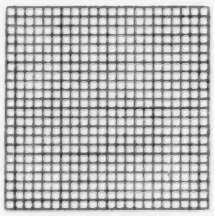

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